

Comparison of relative sensitivities for capacitive and short strip line pickups.

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Introduction. An optimal configuration of sensors for the linac beam position monitor is under investigation now. It can be chosen from a wide variety of electromagnetic pickups: strip line (matched or terminated), magnetic loop, capacitive probe. Note that the basic principle of operation is the same for all these sensors and mentioned above classification is quite arbitrary, i.e. capacitive probe is a short strip line with open ends, magnetic loop is a short strip line with shorted end. "Short strip line" means that its length is much less than the operational wavelengths. In this case the model of a uniform TEM line is not well applicable because edge effects dominate and model of capacitive probe or magnetic loop is more suitable. Directivity and large bandwidth are intrinsic properties of TEM line and can be achieved in configuration with long strip lines only. Due to limitation of the available space in the CCDTL/CCL length of the e/m sensors in the current design limited by about 2cm which is comparable with its azimuthal size and much less than fundamental bunch wavelength of 75cm. So it is not a strip line but a capacitor or a loop depending on termination. In this paper we will compare open and shorted variants from the point of view of the sensitivity to the beam current. The sensitivity to the beam position, linearity and lobe to lobe coupling depends on 2D configuration of BPM cross section and is independent in the first order on pickup length or type of termination.

Capacitive probe. General configuration of a capacitive probe is shown on fig1., ϕ is lobe subtended angle, L is the length of the lobe, R is the vacuum chamber radius. Amplitude of the charge q_λ induced on the lobe by the beam with the amplitude of harmonically modulated longitudinal charge density ρ_λ can be calculated as [1]:

$$q_l = r_l(t) \cdot \frac{\sin(pL/\lambda)}{p/\lambda} \cdot \frac{1}{I_0(g)} \cdot \frac{f}{2p},$$

where λ is wavelength of the charge density harmonics, I_0 is modified Bessel function, and $g=2\pi R/\beta\gamma\lambda$ includes effects of beam velocity β and relativistic factor γ .

This charge produces the current I_λ in the lobe connection:

$$I_l = \dot{q}_l = w \cdot r_{l0} \cdot \frac{\sin(pL/\lambda)}{p/\lambda} \cdot \frac{1}{I_0(g)}$$

The equivalent impedance of the capacitive probe is

$$Z = \frac{R}{\sqrt{1 + (\omega RC)^2}},$$

where R is input impedance of the amplifier and C is probe capacity. Then the output voltage is

$$U_1 = I_1 \cdot Z = \omega \cdot r_{10} \cdot \frac{\sin(\frac{\rho L}{I})}{\rho/I} \cdot \frac{1}{I_0(g)} \cdot \frac{R}{\sqrt{1 + (\omega RC)^2}} \cdot \frac{f}{2p}.$$

If $L \ll \lambda$ we can substitute sin by its argument. $\rho_{\lambda 0}$ can be derived from the beam current I_b as $\rho_{\lambda 0} = I_b / \beta c$, then

$$U_1 = \frac{I_b \cdot L}{b \cdot c} \cdot \frac{1}{I_0(g)} \cdot \frac{\omega R}{\sqrt{1 + (\omega RC)^2}} \cdot \frac{f}{2p}.$$

Choosing parameters of the circuit such that $\omega RC \ll 1$ or $\omega RC \gg 1$ we have two expressions for the probe sensitivity:

$$U_1 = \frac{I_b \cdot L}{b \cdot c} \cdot \frac{\omega RC}{I_0(g)} \cdot \frac{1}{C} \cdot \frac{f}{2p} \quad \text{for small input impedance } (\omega RC \ll 1),$$

$$U_1 = \frac{I_b \cdot L}{b \cdot c} \cdot \frac{1}{I_0(g)} \cdot \frac{1}{C} \cdot \frac{f}{2p} \quad \text{for large input impedance } (\omega RC \gg 1).$$

It is evident that value of the second expression is always larger than value of the first one therefore large input impedance is preferable for the capacitive probe. In our case of $\omega = 2\pi \cdot 402.5$ MHz we have to provide $R[\Omega] \gg 395/C[\text{pF}]$. In practice $C \geq 10\text{pF}$ therefore $R=50\Omega$ is a good choice already.

Magnetic loop (shorted short strip line). Voltage induced on the matched end of a stripline which second end is short-circuited is described by expression:

$$U_1 = I_b \cdot Z_0 \cdot \sin\left(\frac{\omega L}{c}\right) \cdot \frac{1}{I_0(g)} \cdot \frac{f}{2p},$$

where Z_0 is characteristic impedance of the strip line. If line is short we can rewrite it as

$$U_1 = \frac{I_b \cdot L}{c} \cdot \frac{1}{I_0(g)} \cdot \omega Z_0 \cdot \frac{f}{2p}.$$

Strip line with resonant length. Voltage induced on the matched end of a strip line which second end is short circuited and length is defined by condition of resonance $\omega L_0/c = \pi/2$ is described by expression:

$$U_1 = I_b \cdot Z_0 \cdot \frac{1}{I_0(g)} \cdot \frac{f}{2p}$$

Conclusion. The ratio of the capacitive probe output to magnetic loop output is

$$\frac{U_{cap}}{U_{loop}} = \frac{1}{C \cdot b} \cdot \frac{1}{wZ_0} = \frac{10^{12}}{C[pF] \cdot b} \cdot \frac{10^{-6}}{2p \cdot 402.5[MHz]} = \frac{7.91}{C[pF]} \cdot \frac{1}{b}$$

It is proportional to $1/\beta$ as we could expect from the nature of electric coupling in the case of capacitive probe and magnetic coupling in the case of loop.

The ratio of the capacitive probe output to strip line with resonant length output is

$$\frac{U_{cap}}{U_{res}} = \frac{L}{c \cdot b} \cdot \frac{1}{C \cdot Z_0} = \frac{L}{L_0} \cdot \frac{p}{2} \cdot \frac{1}{b} \cdot \frac{1}{C} \cdot \frac{1}{wZ_0} = \frac{L}{L_0} \cdot \frac{10^{12}}{C[pF] \cdot b} \cdot \frac{10^{-6}}{4 \cdot 402.5[MHz] \cdot 50\Omega} = \frac{L}{L_0} \cdot \frac{12.4}{C[pF]} \cdot \frac{1}{b}$$

Calculated ratio of sensor sensitivities are shown in the figure 1a,b for the case of $C=10pF$, $L=2cm$. As one can see the strip line with resonant length (19cm) is the best choice for beam energies above 10MeV. Capacitive probe outperforms magnetic loop at energies below 600 MeV. Note that at 2.5MeV the capacitive probe is twice more sensitive than even the resonant strip line that can be important for choice of the sensor for the phase measurements. Possible drawback of capacitive probe is the dependence of its sensitivity on beam velocity but it doesn't affect position measurement.

References. [1] Monitoring of particle beams at high frequencies. J.Cuperus, NIM 145(1977) 219-231

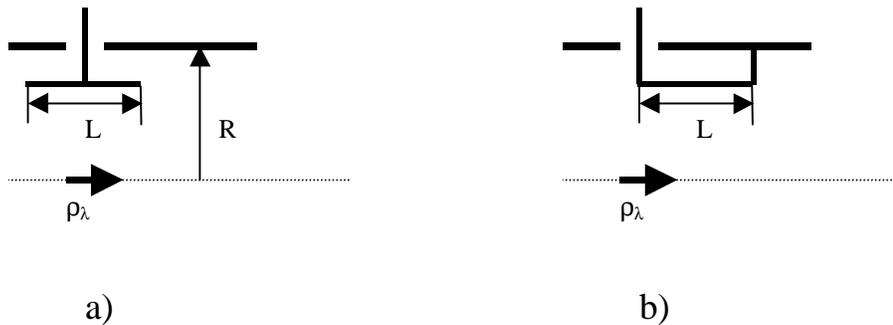


Figure 1. Schematic view of capacitive probe (a) and magnetic loop (b).

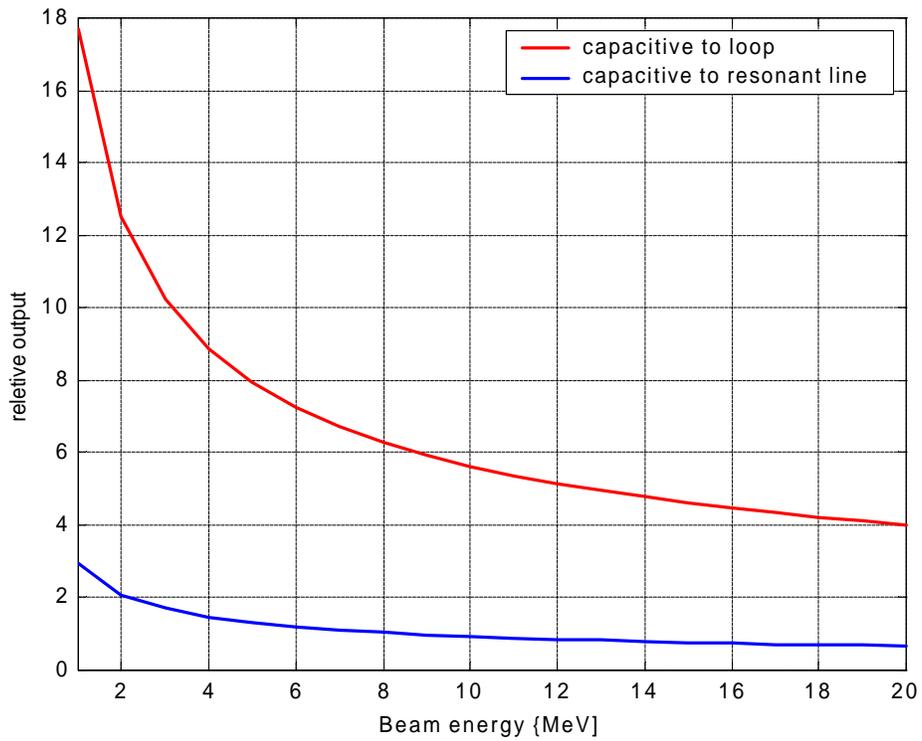
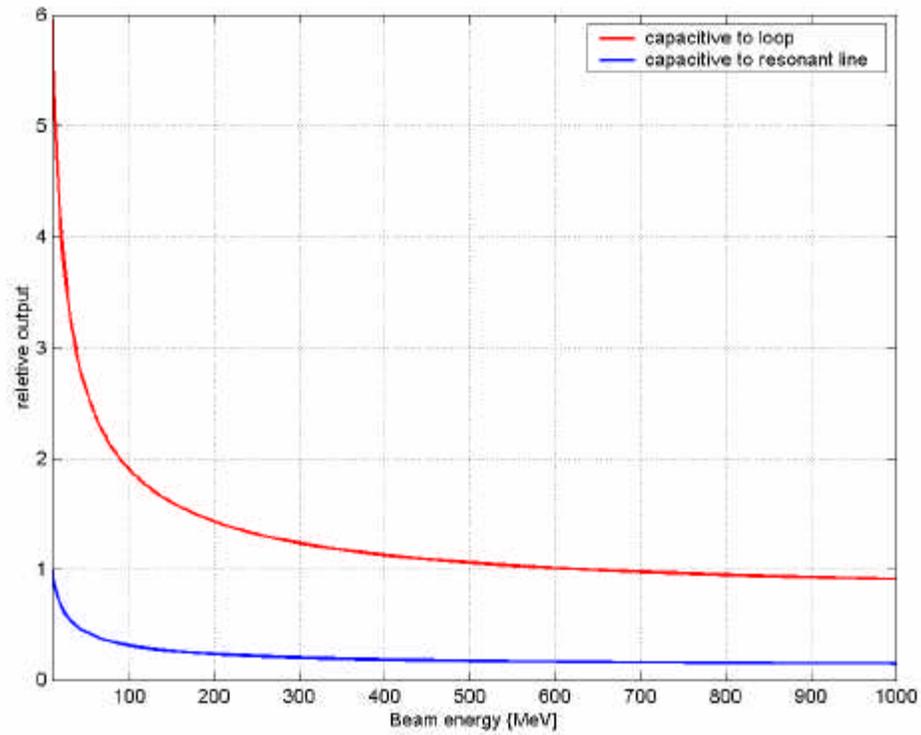


Figure 2. The sensitivity of the resonant strip line (blue line) and the magnetic loop (red line) relative to sensitivity of the capacitive probe.