

# X-ray Photon Correlation Spectroscopy

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NSLS-II, Brookhaven National Laboratory  
National School on Neutron and X-ray Scattering, June 2019



- **Introduction**

- Why (opportunities for mesoscale science) and How (coherence and speckles)
- Speckle fluctuations, dynamics
- Speckle Statistics

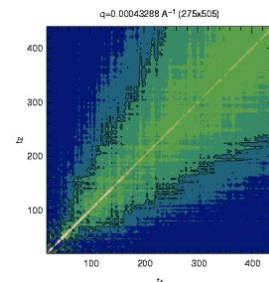
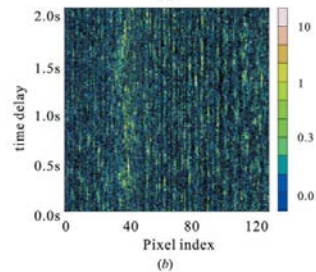
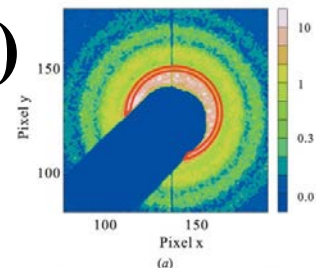
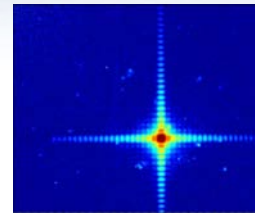
- **X-ray Photon Correlation Spectroscopy (XPCS)**

- Time autocorrelation functions, equilibrium dynamics
- Signal-to-Noise
- Two-time correlation functions, non-equilibrium dynamics
- Higher order correlation functions, dynamical heterogeneities
- X-ray Speckle Visibility Spectroscopy
- A mini user guide to XPCS

- **XPCS examples**

- Dynamics of concentrated hard-sphere suspensions. Is there a colloidal glass transition?
- “Anomalous” relaxations in “jammed” systems

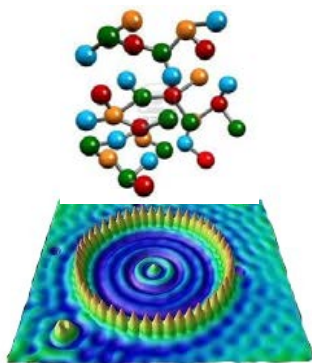
- **Conclusions**



# The Next “Big Thing”

- Opportunities for “Mesoscale Science” DOE BESAC report Sept 2012  
<http://www.meso2012.com>

“Nano” nm



Reductionist Science  
 “Theory of Everything”

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

Not practical....

**Biomembrane**

**Kinesin**

**Cu<sub>3</sub>Au**

**Polymer blend**

**Colloids**

**SP40 virus**

“Meso”  
 in-between

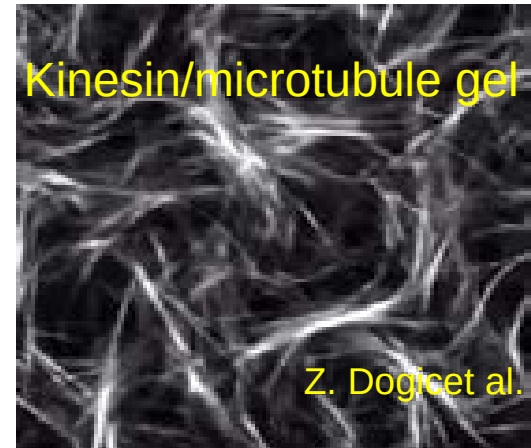
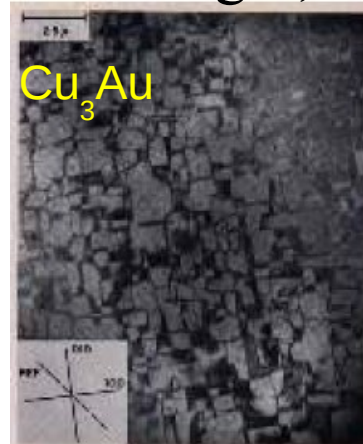
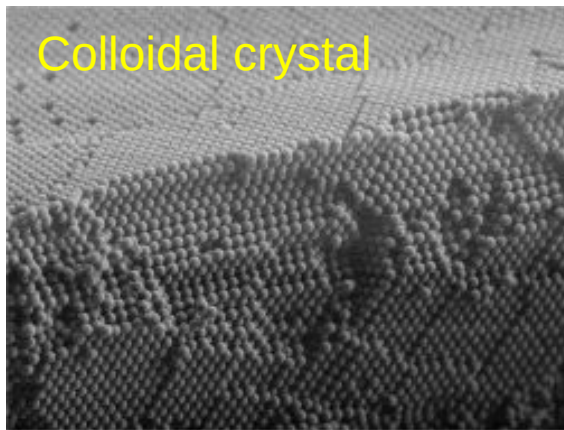
“Macro”  
 μm, mm, m, ...  
 hydrodynamics  
 rheology



# “More is Different”

P.W. Anderson, *Science* 177, 393 (1972)

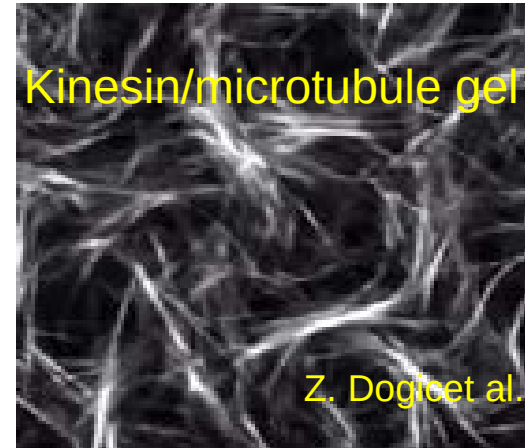
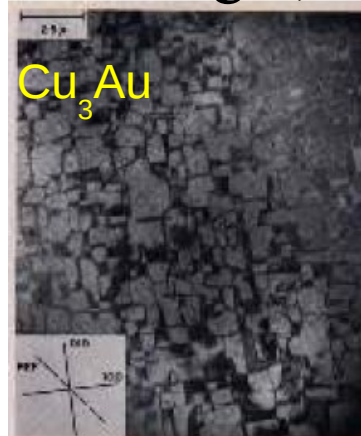
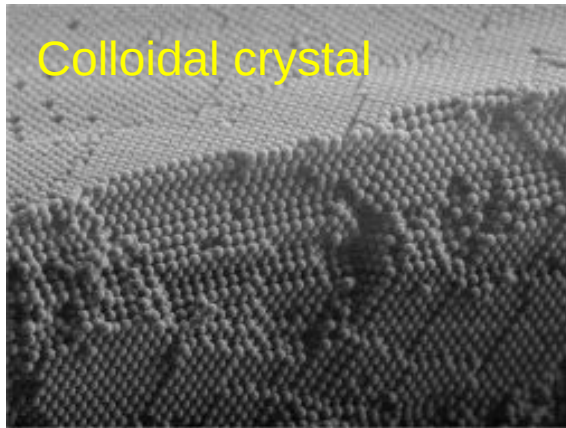
- Most *macroscopic properties* of *complex disordered materials* emerge at the *mesoscale* (nm to  $\mu\text{m}$ ):
  - Mesoscale structure: defects, grain size, macromolecule shape/size, entanglement length, ...



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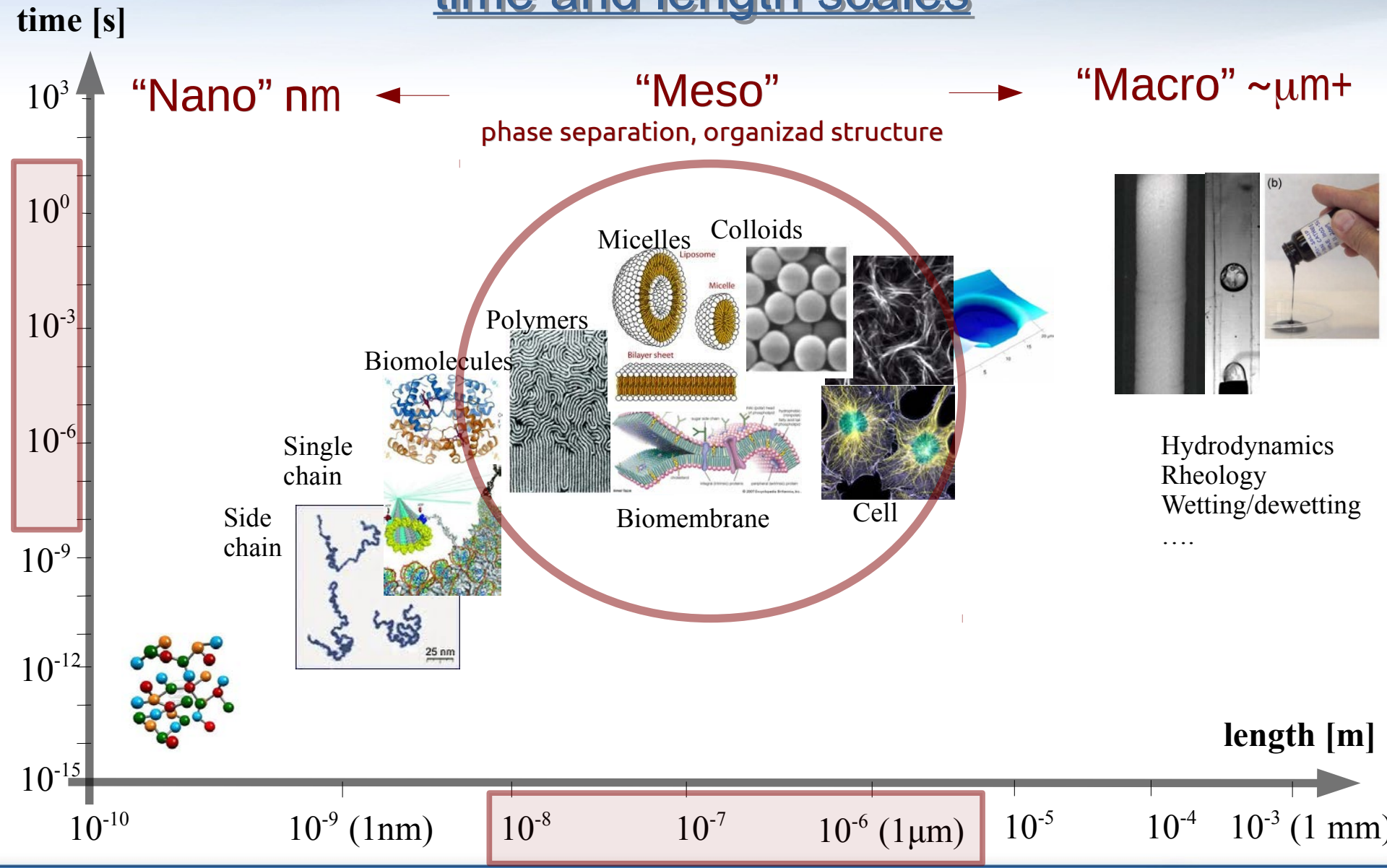
**But things are not static !**

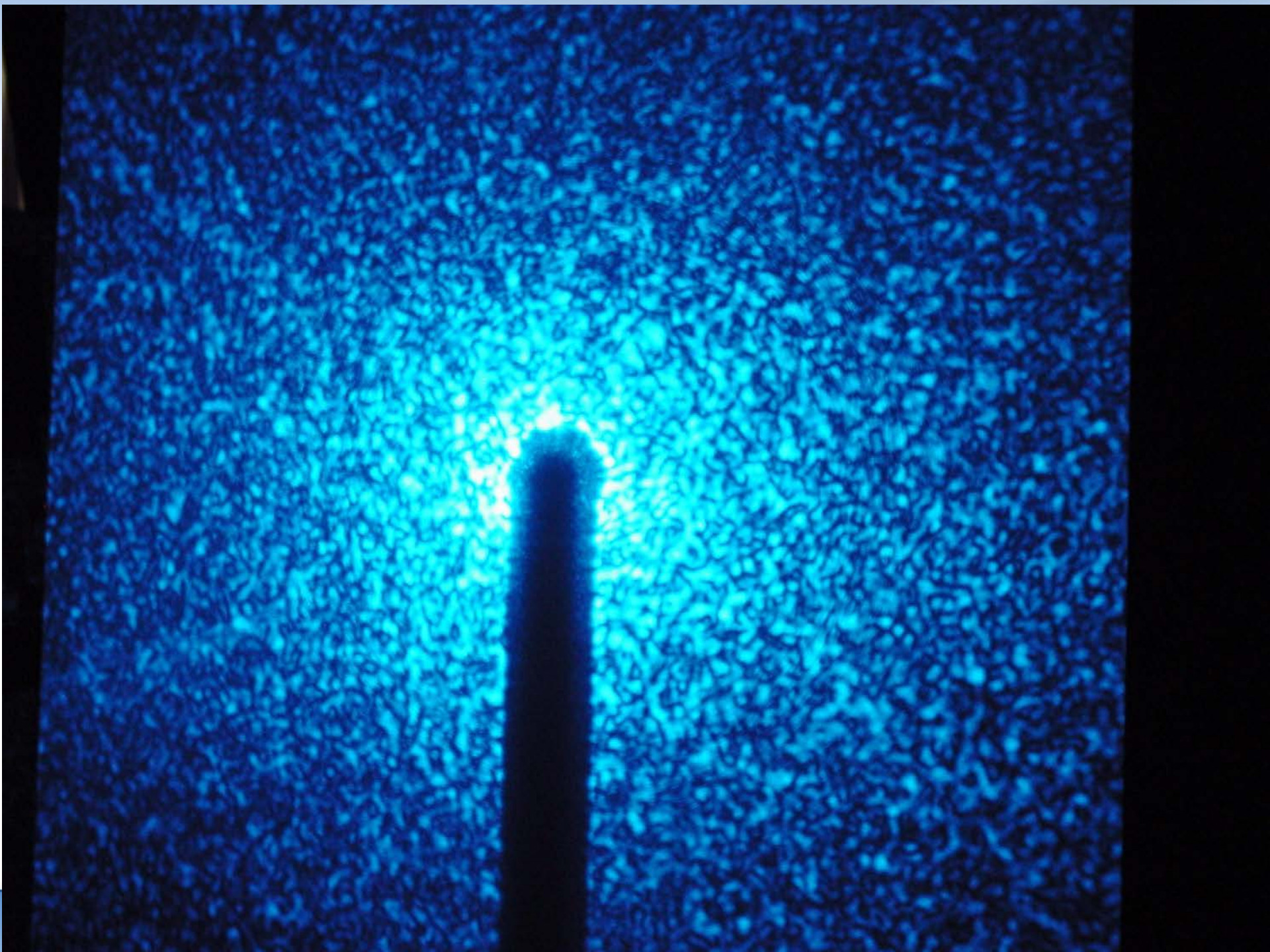
- Mesoscale Dynamics

Z. Dogic (Brandeis Univ.)  
Dynamics of bundled active networks

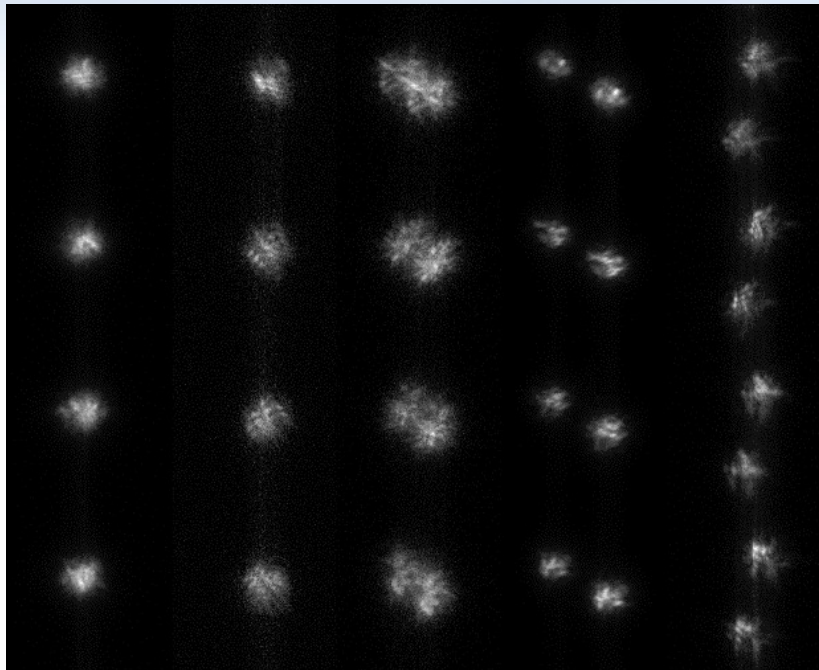


# Dynamics of Materials (soft- and bio-): time and length scales





# Speckle



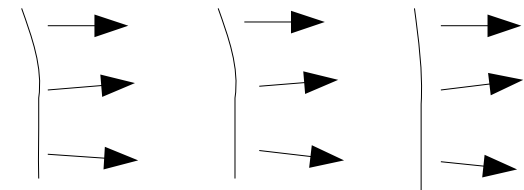
Images of Binary Stars at  
different degrees

Of separation in the  
WIYN Telescope

Matthew F. Hoffmann

<http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html>

- Stars (far away) = nearly coherent “point-like” sources
- Fluctuations in the atmosphere create speckle

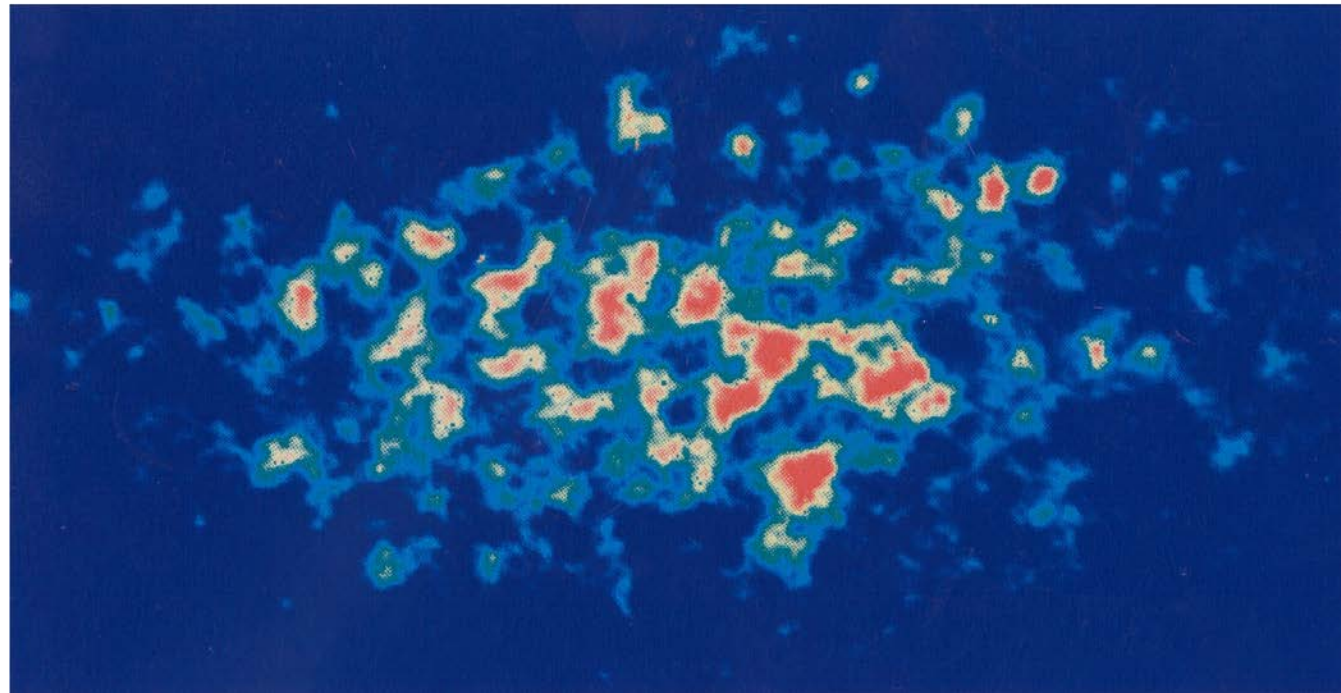
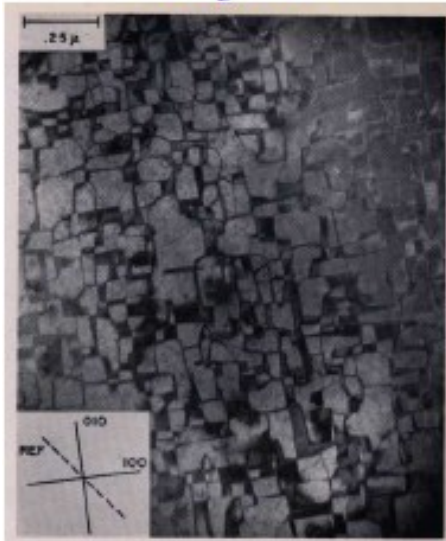




# Speckles with (partially) coherent X-rays

## Speckles from $\text{Cu}_3\text{Au}$

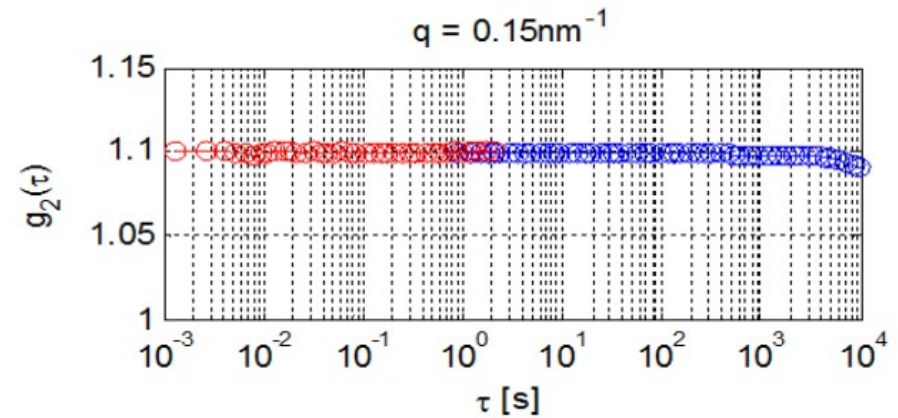
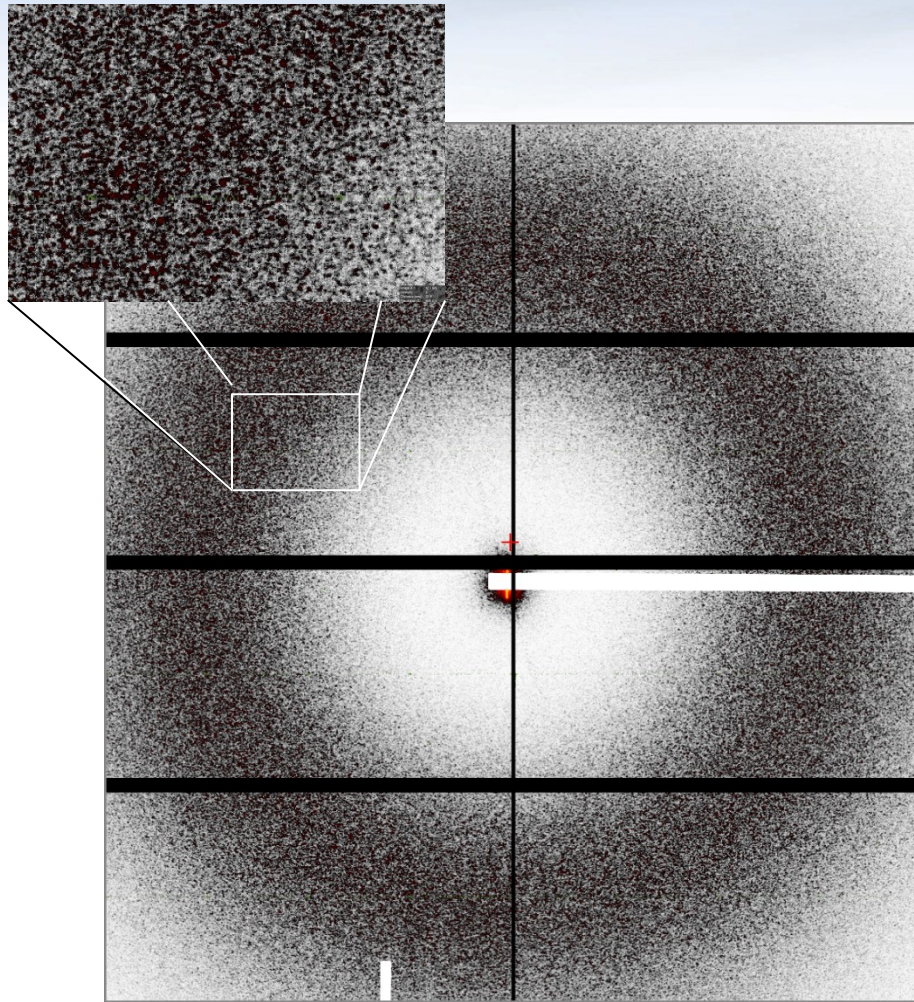
$\text{Cu}_3\text{Au}$



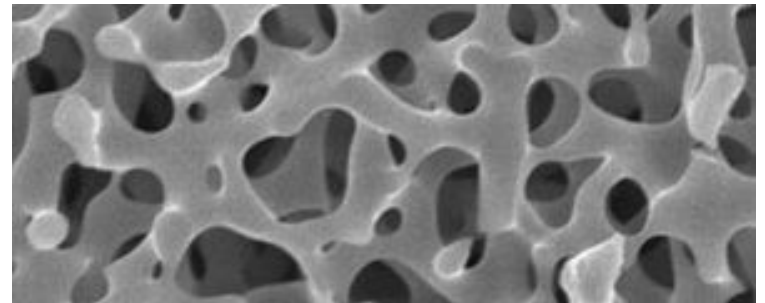
Recorded at X25, NSLS on Kodak film

M. Sutton, *et al. Nature* **352**, 608 (1991)

# X-ray Speckles (Static!)



Correlation functions  $g^{(2)}(q, \tau)$  measured from a CoralPor® static sample show excellent instrument stability.

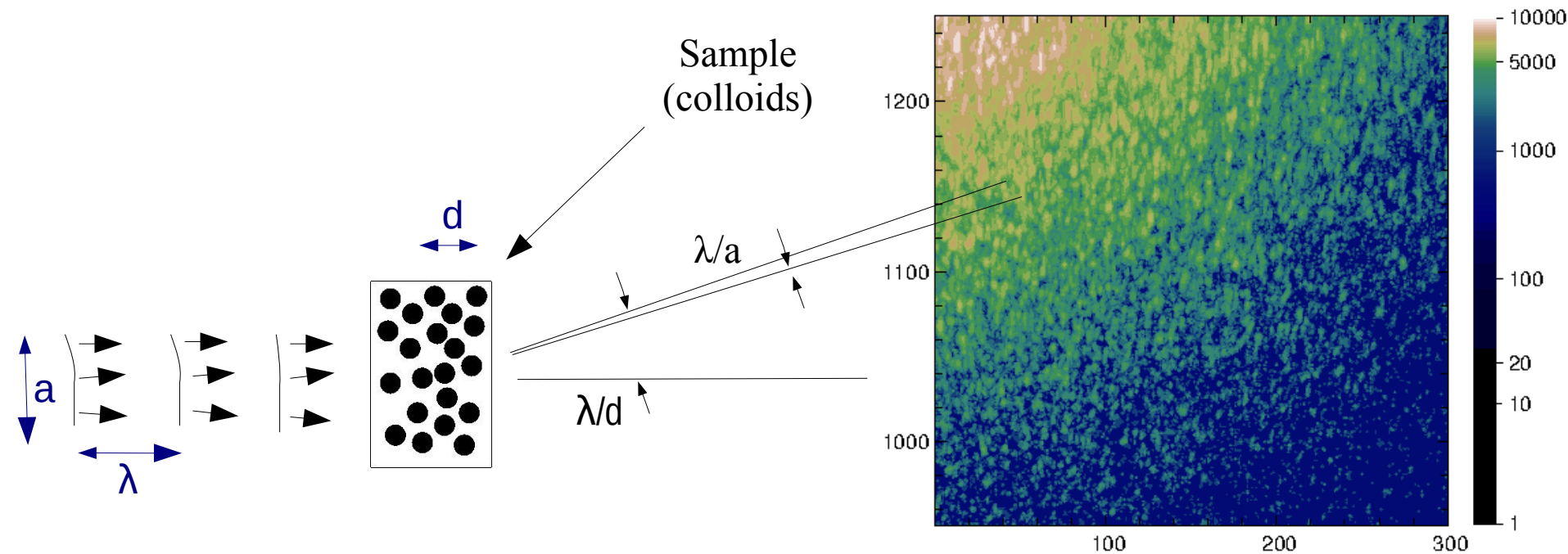


[www.schott.com](http://www.schott.com)  
m

250mA top-off,  $1.5 \times 10^{11}$  ph/s in  $10 \times 10 \mu\text{m}^2$ ; total dose = 101 seconds of “full flux”  
Note: decay at  $\sim 5 \times 10^3$  seconds due to ‘beam damage’

# Speckles with (partially) coherent X-rays

## Speckles from colloidal suspensions

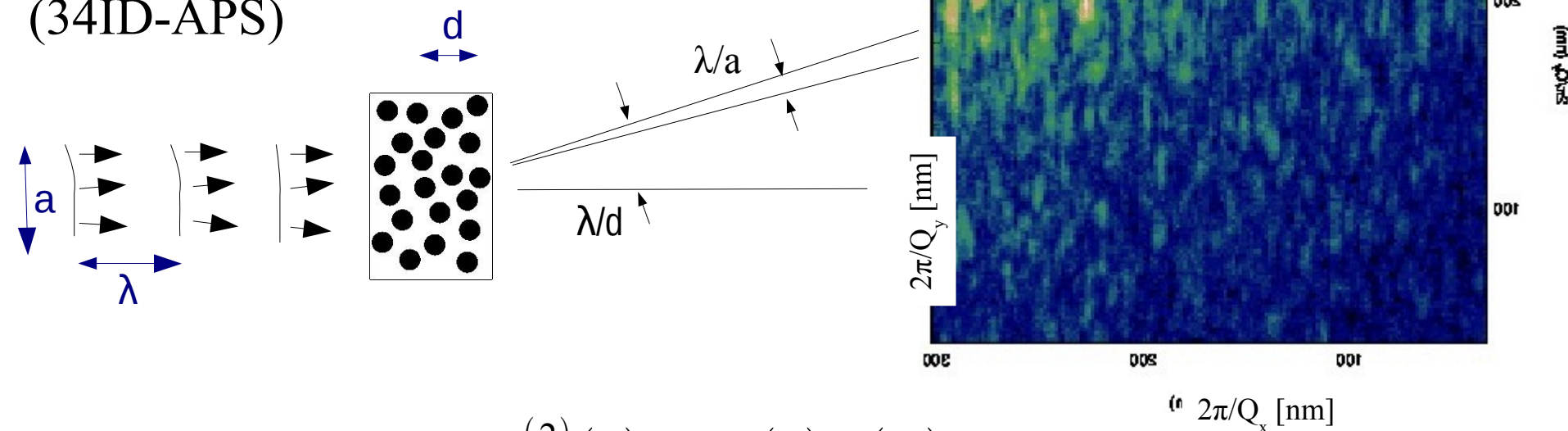


Measured at 34ID with a CCD detector

# Speckle Fluctuations & Dynamics

- At high brightness light sources (APS, ESRF, Petra-III, NSLS-II ...) it is possible to measure dynamics by recording “speckle movies”

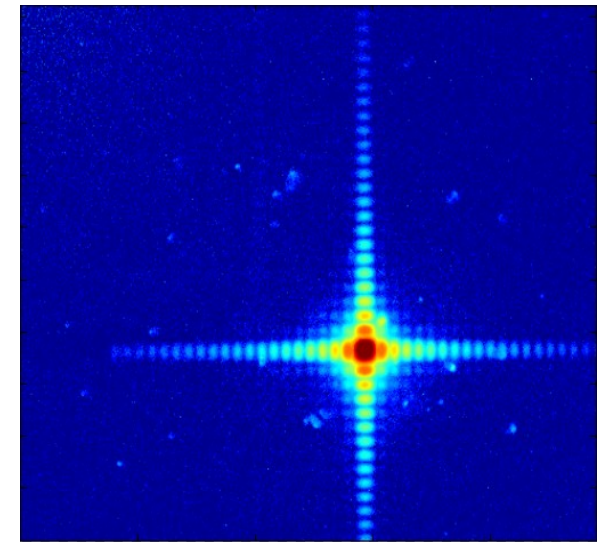
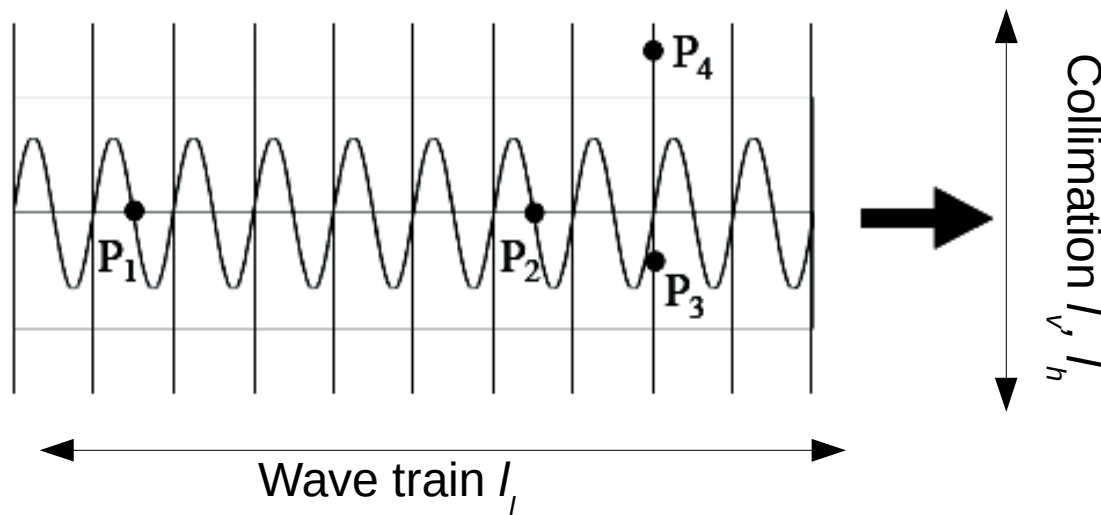
## Partially Coherent X-rays (34ID-APS)



$$g^{(2)}(t) \propto \langle I(t) I(0) \rangle$$

# Mini-introduction to coherence

- Coherence = ability to create interference fringes w. good contrast
  - i.e. exists within a region where the phase difference between any pair of points is well defined and constant in time
  - Transverse coherence:  $\Delta\Phi(P3:P4)$
  - Longitudinal(temporal) coherence:  $\Delta\Phi(P1:P2)$



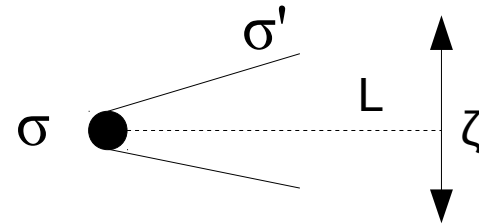
Malcolm Howells, Lecture Notes, ESRF 2007

L. Wiegart, CHX, NSLS-II

# Transverse coherence

- Ideal *coherent* (Gaussian) source:
  - a source cannot be arbitrarily small and arbitrarily well collimated at the same time (diffraction limit)

$$\sigma \cdot \sigma' \simeq \frac{\lambda}{4\pi}$$



- A transverse coherence length (@ distance  $L$  from the source) can then be defined as:

$$l_{h,v} = \frac{\lambda L}{4\pi \sigma_{h,v}}$$

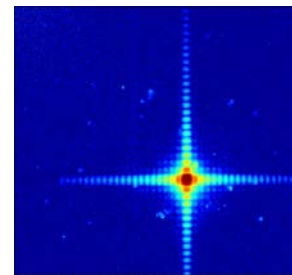
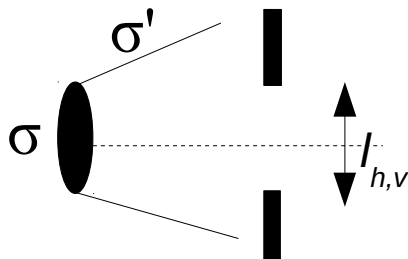
# Transverse coherence

- Real Source:

- The degree of coherence is determined by the phase space volume  $\sigma\sigma'$ ; “Heisenberg's inequality”:

$$\sigma \cdot \sigma' \geq \frac{\lambda}{4\pi}$$

- “Liouville's theorem”: the phase space is conserved by propagation, (ideal) crystal optics, (ideal) focusing, etc.
- To obtain a more coherent beam (at the expense of flux!), the phase space can be limited/reduced by collimation (a set of slits)



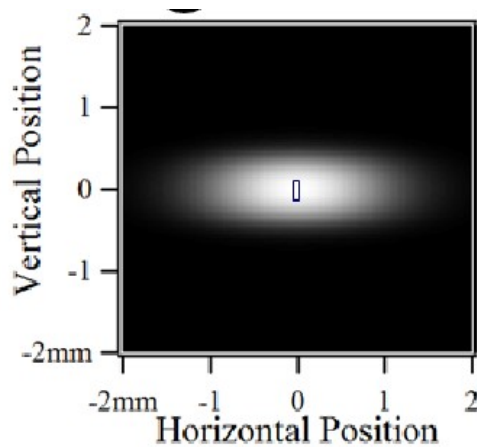
# Coherence of (NSLS-II) Synchrotron Sources

- Real Source:

- Number of coherent modes:

$$\sigma \cdot \sigma' = N \frac{\lambda}{4\pi}, N \geq 1$$

- E.g. IVU20 undulator source at CHX, NSLS-II



E (keV)	6	8	10	12	16
$\sigma_h$ ( $\mu\text{m}$ )	34.3	34.2	34.1	34.2	34.2
$\sigma_h'$ ( $\mu\text{rad}$ )	18.3	18.3	18.0	18.2	18.2
$\sigma_v$ ( $\mu\text{m}$ )	8.8	8.0	7.5	7.6	7.4
$\sigma_h'$ ( $\mu\text{rad}$ )	8.5	8.2	7.7	8.1	8.0
$M_h$	38.2	50.7	62.2	75.7	94.6
$M_v$	4.5	5.3	5.8	7.5	9.0



# Longitudinal coherence

- Longitudinal (temporal) coherence:

$$\frac{\delta \lambda}{\lambda} \approx \frac{1}{N}, l_l = \lambda N \quad l_l \approx \frac{\lambda^2}{\delta \lambda}$$

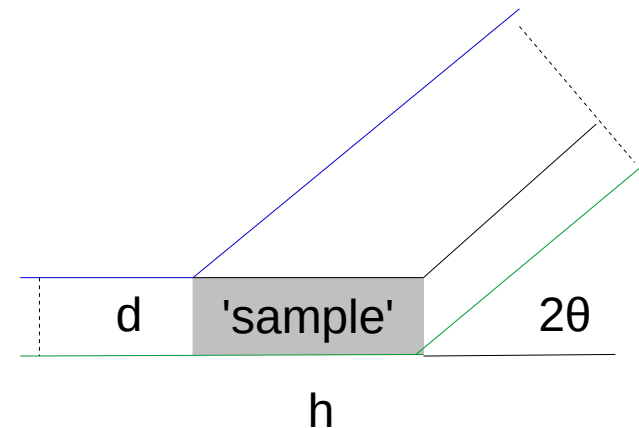
- Experimental requirement:

$$\text{max optical path diff.} < l_l$$

- In a transmission geometry

- Sample thickness  $h$ , beam size  $d$

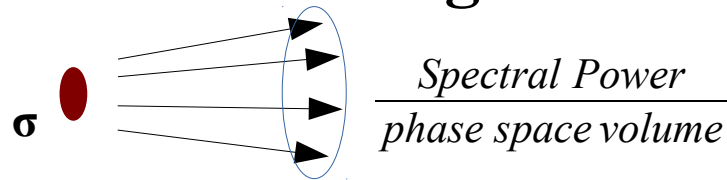
$$h \sin^2(2\theta) + d \sin(\theta) \leq l_l$$



A. Madsen, A. Fluerasu, B. Ruta, Structural Dynamics of Materials probed by X-ray Photon Correlation Spectroscopy, Springer, 2014

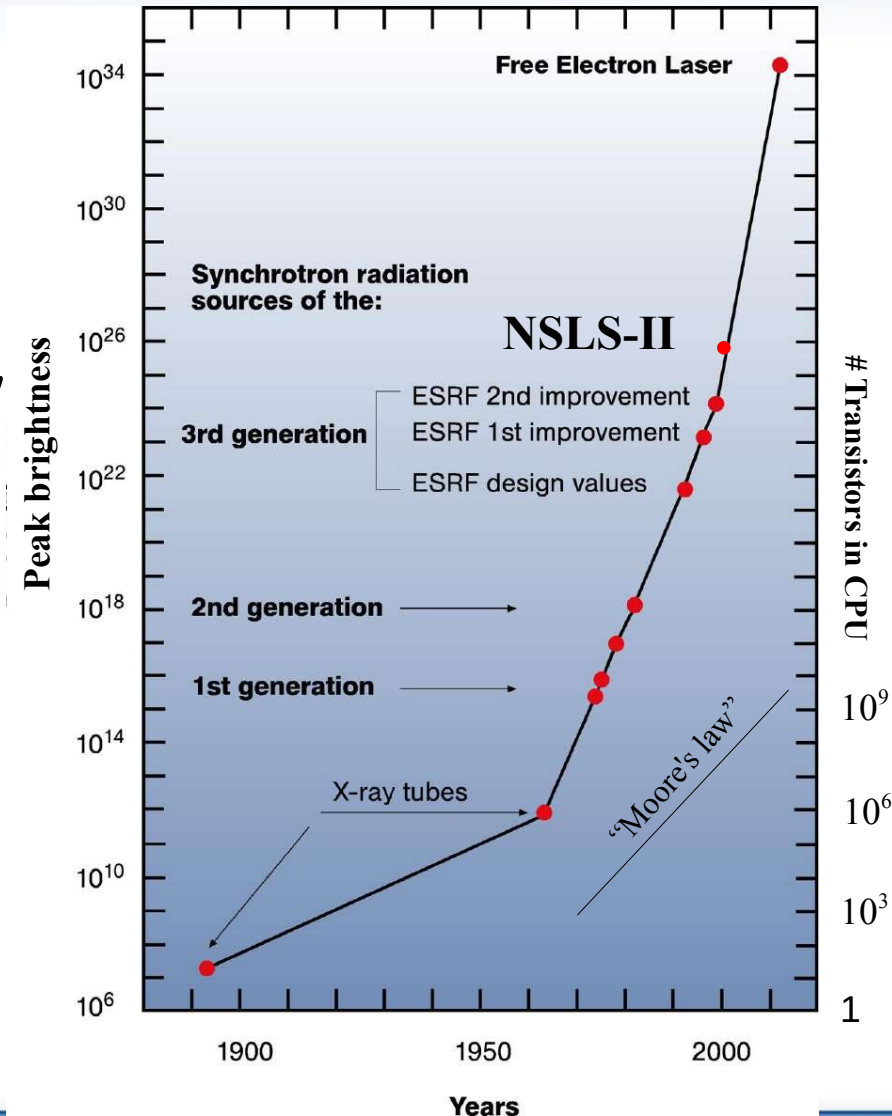
# Synchrotron Source Brightness

- Key for XPCS: **Brightness**



*Brightness=Coherence*  
*increased faster than Moore's law!!*

- Coherent Flux  $I \propto B \lambda^2$
- CHX, NSLS-II ( $\sim 10$  keV)  
 $B \sim 10^{21}$  ph/s/%bw/mm<sup>2</sup>/mrad<sup>2</sup>  
 $I \sim 10^{11}$  ph/s



# Correlation Functions

- Coherence → measures dynamics

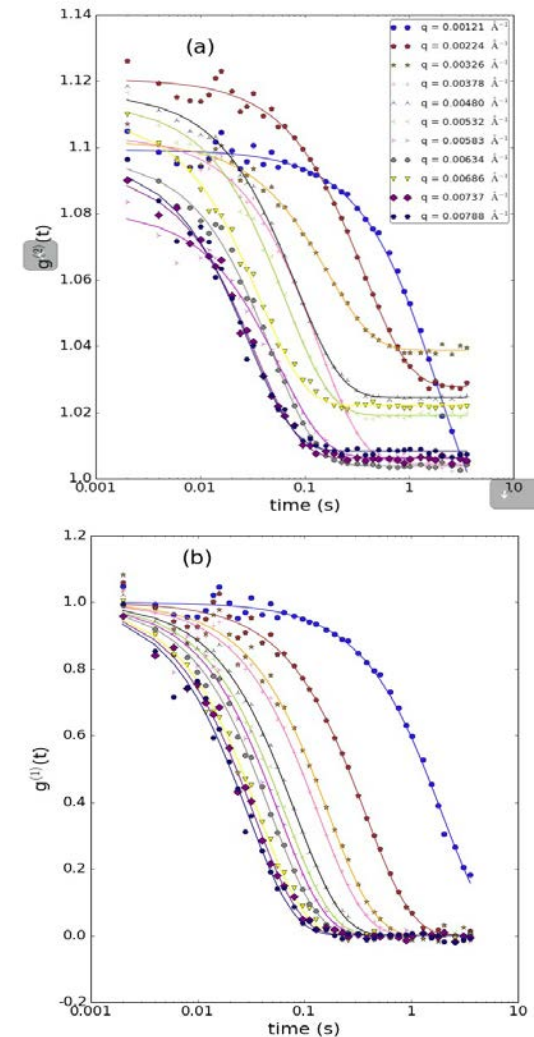
$$\langle I(q, t) I(q, t + \delta t) \rangle = \langle I(q) \rangle^2 + \beta(q) (\dots) |S(q, t)|^2$$

- Intensity autocorrelation function, dynamic structure factor & Siegert relationship:

$$g^{(2)}(q, t) = \frac{\langle I(q, t) I(q, t + \delta t) \rangle}{\langle I(q) \rangle^2} = 1 + \beta(q) \left| \frac{S(q, t)}{S(q, 0)} \right|^2$$

- Intermediate Scattering Function

$$g^{(1)}(q, t) = \left| \frac{S(q, t)}{S(q, 0)} \right| \propto \iint \rho_n(q) \rho_m(q) \exp(iq[r_n(0) - r_m(t)])$$



# Correlation Functions

- Signal-to-noise (of  $g^{(2)}$ ) – it's complicated!!

$$R_{sn} = K (T \tau \Omega_x \Omega_z)^{1/2} \Sigma W \exp(-W \Lambda) \tilde{B} (\Delta E / E) r_{snx} r_{snz}$$

K = detector efficiency

T = total experiment duration

$\tau$  = accumulation time

$\Omega$  = angle subtended by Q of interest

$\Sigma$  = scattering cross section per unit volume

W = sample thickness

$\Lambda$  = 1/attenuation length

B = source brilliance

$\Delta E/E$  = normalized energy spread

r = factor depending on source size, pixel size, and slit size

- SNR  $\sim B\tau^{1/2} \dots$
- Need an area det
- $\sim$ small pixels
- fast frame rates

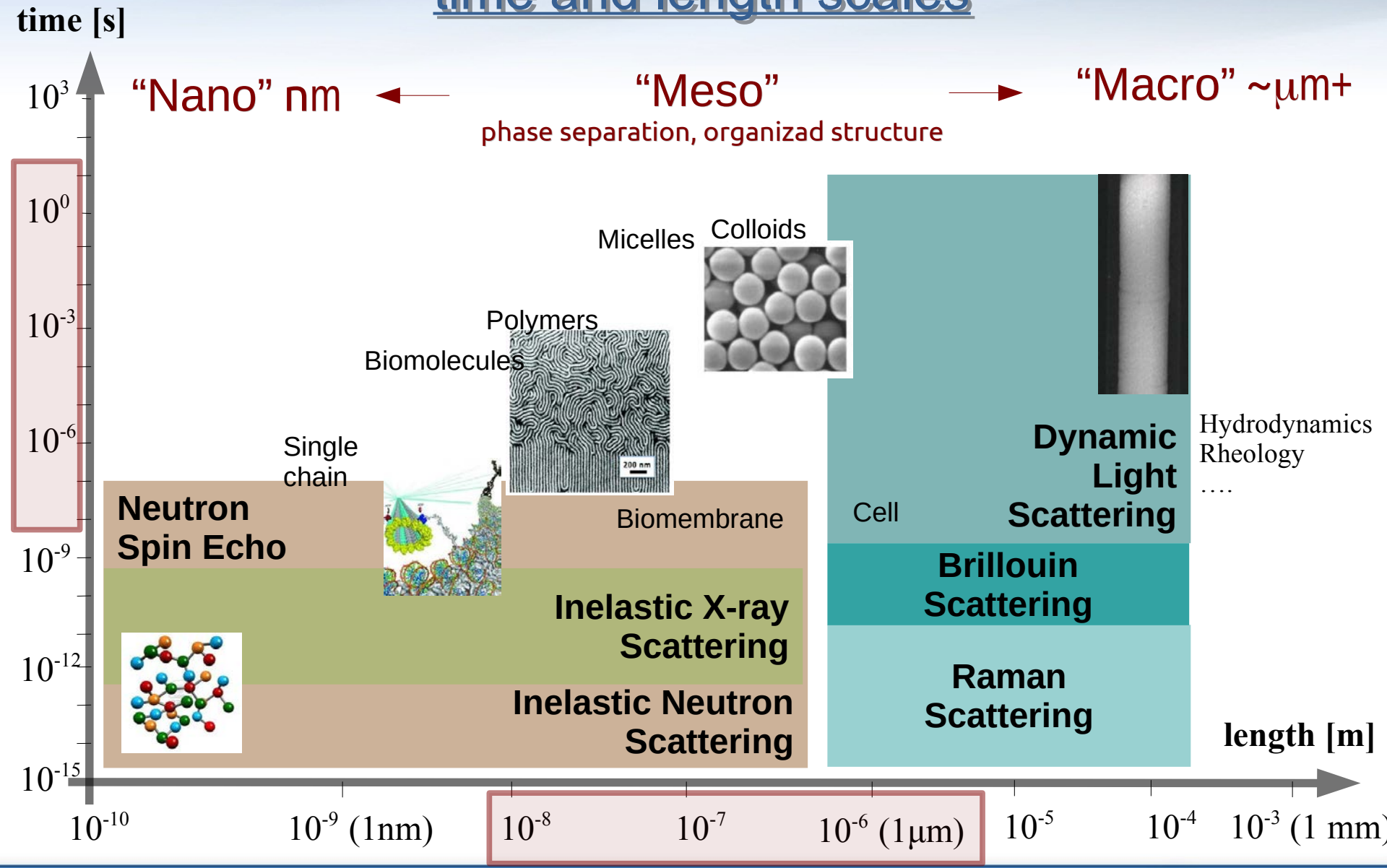


Eiger 1M detector  
(Dectris)

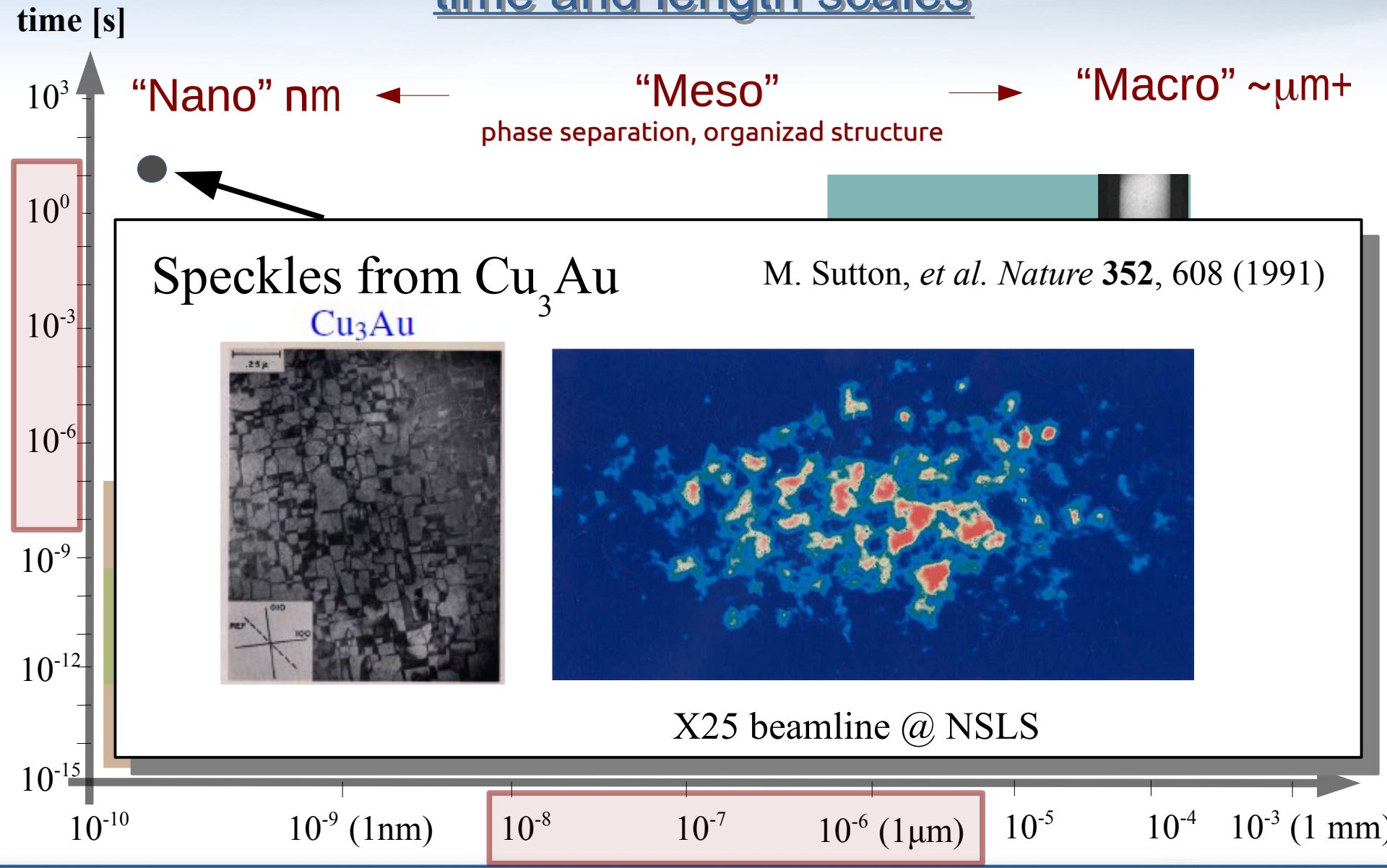
Lumma *et al.* *Rev. Sci. Instrum.* 71, 3274 (2000)

Jackeman *et al.* *J. Phys. A*, 5, 517 (1971)

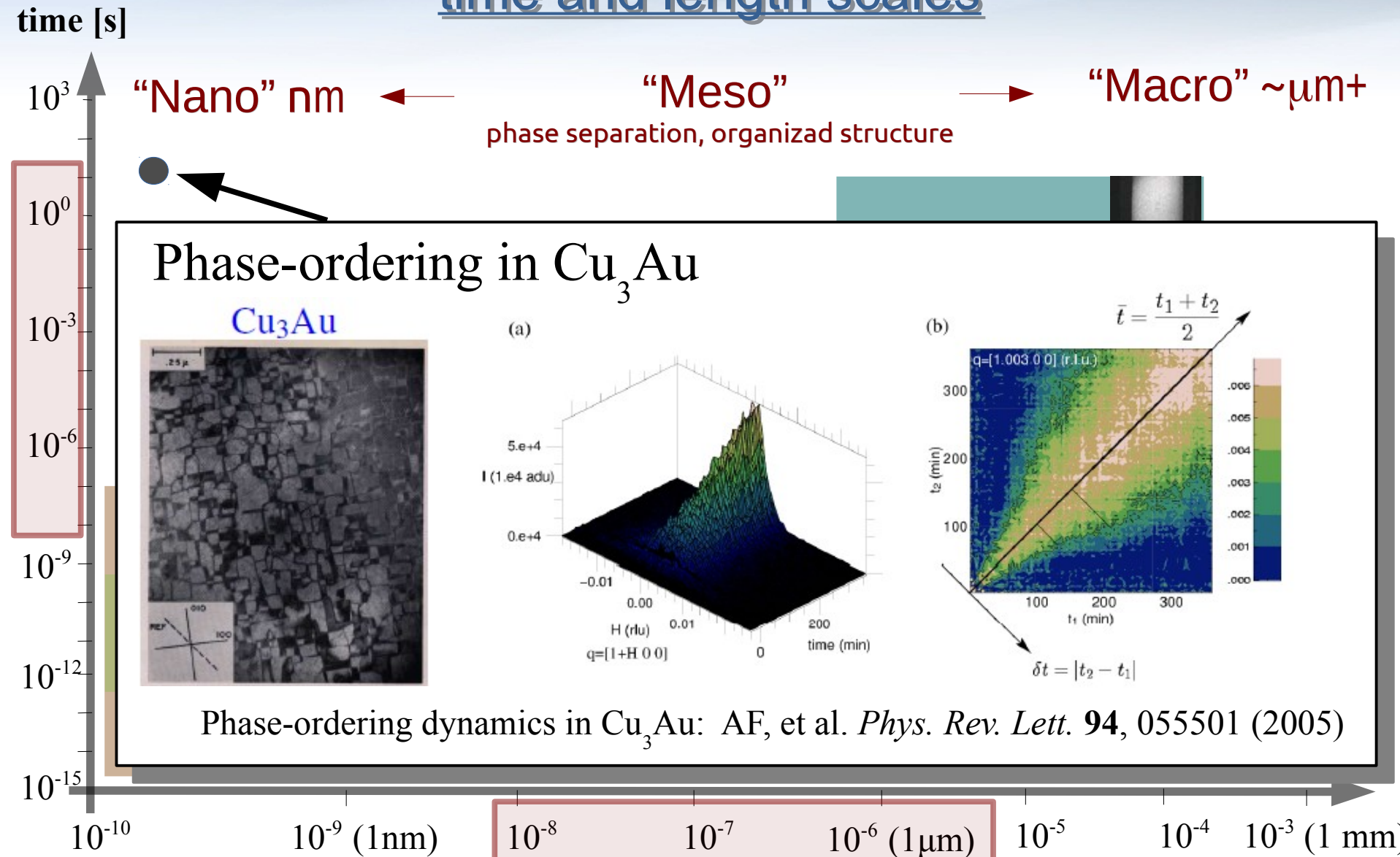
# Dynamics of Materials (soft- and bio-): time and length scales



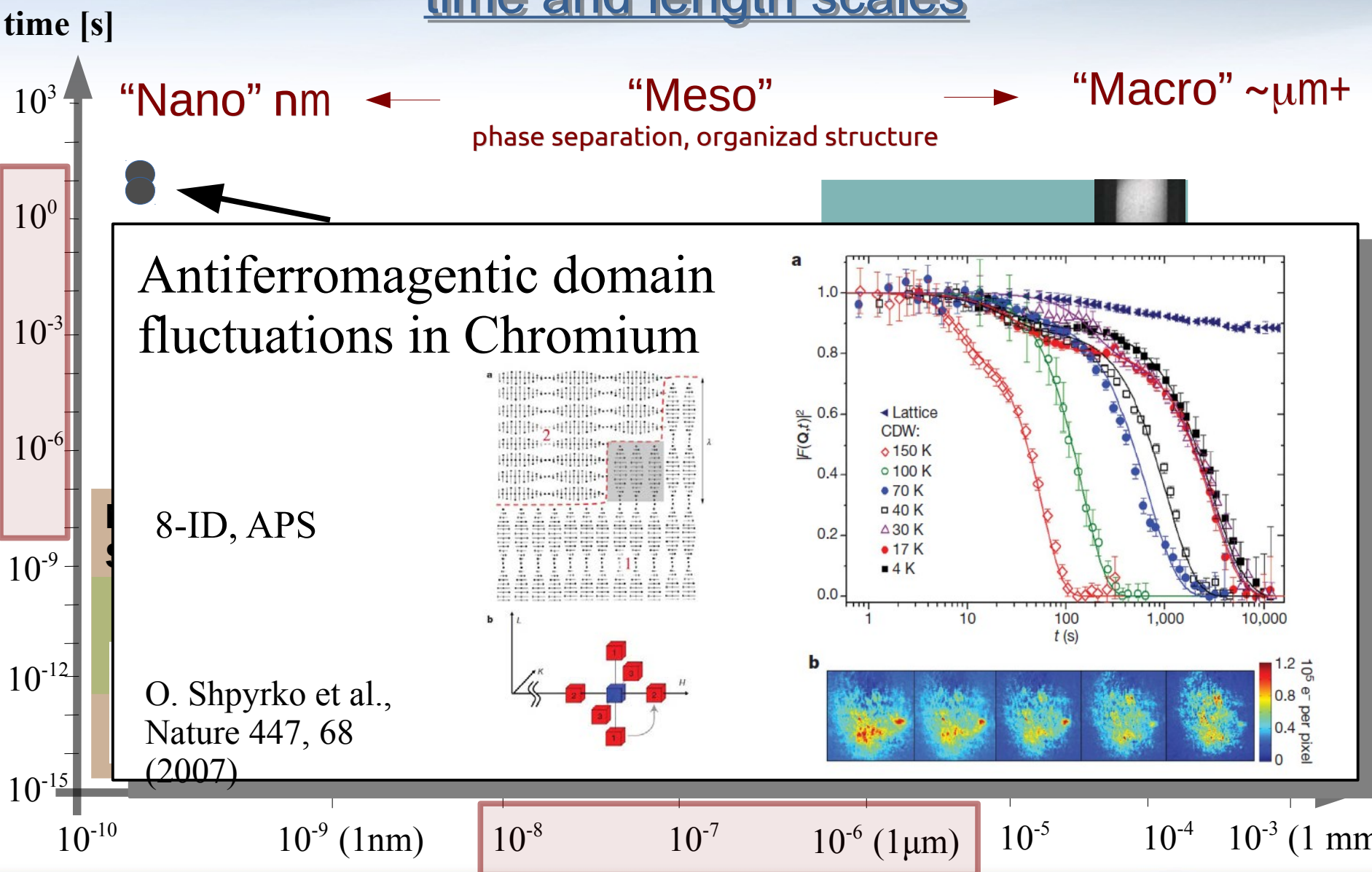
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# Dynamics of Materials (soft- and bio-): time and length scales

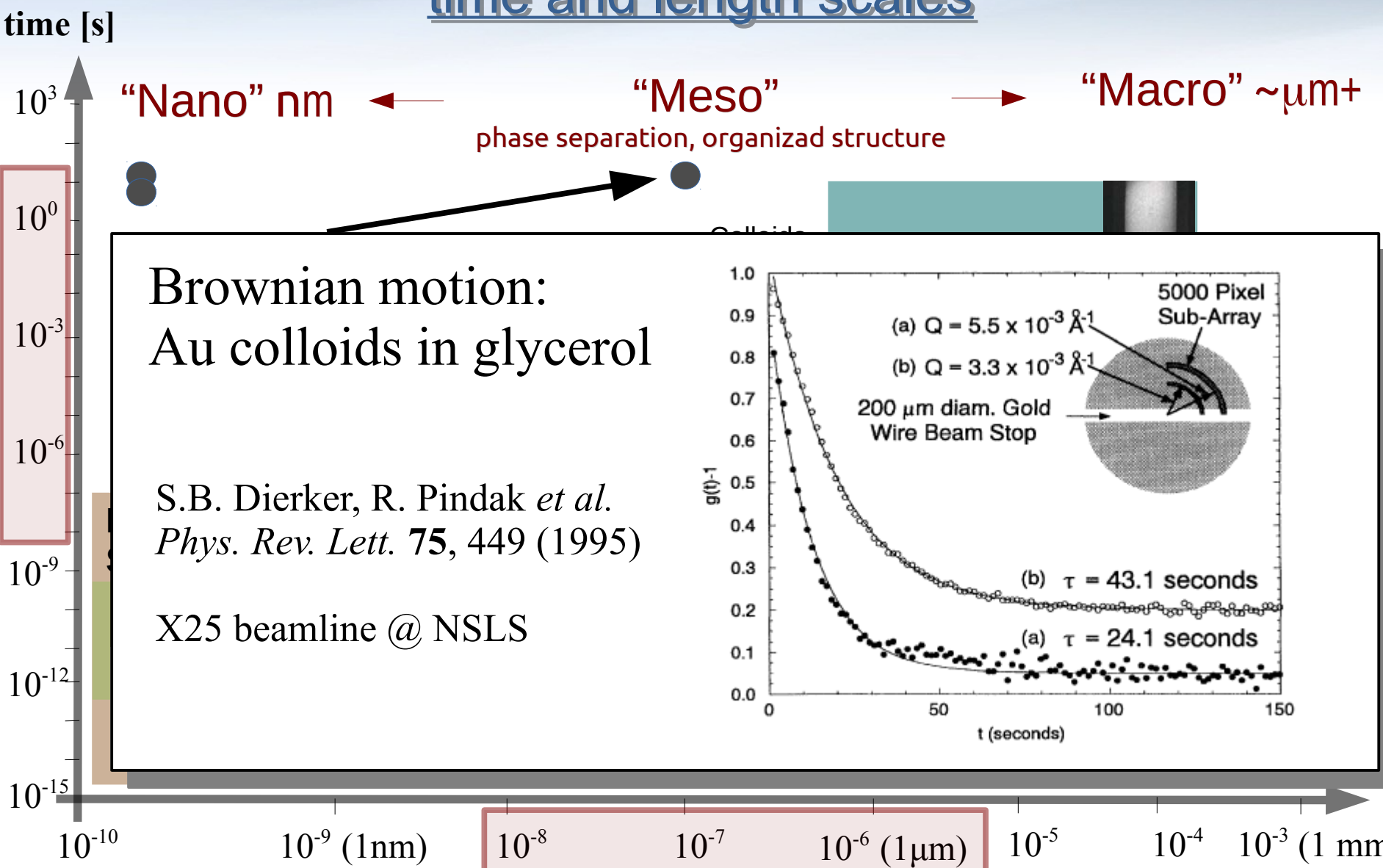


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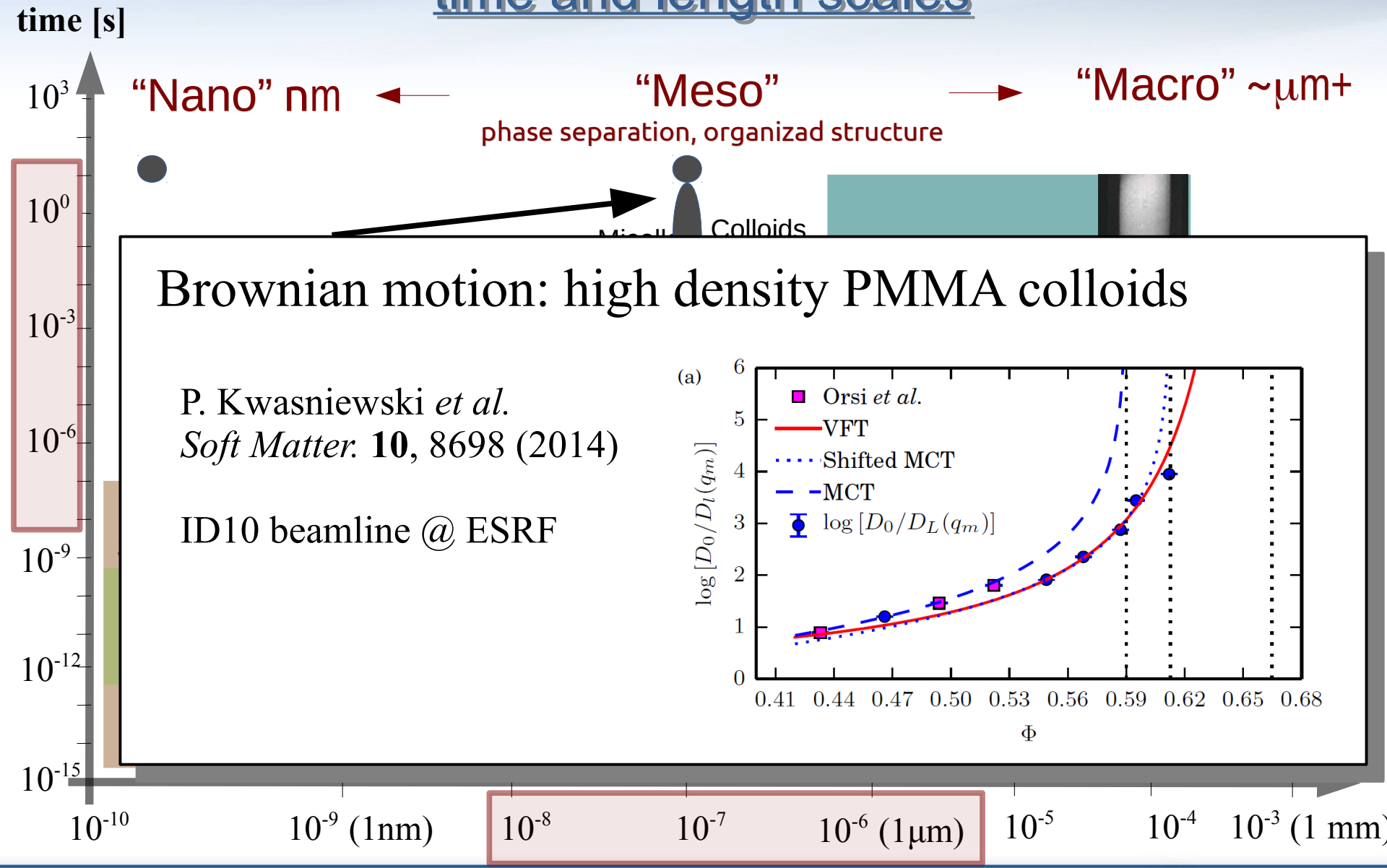




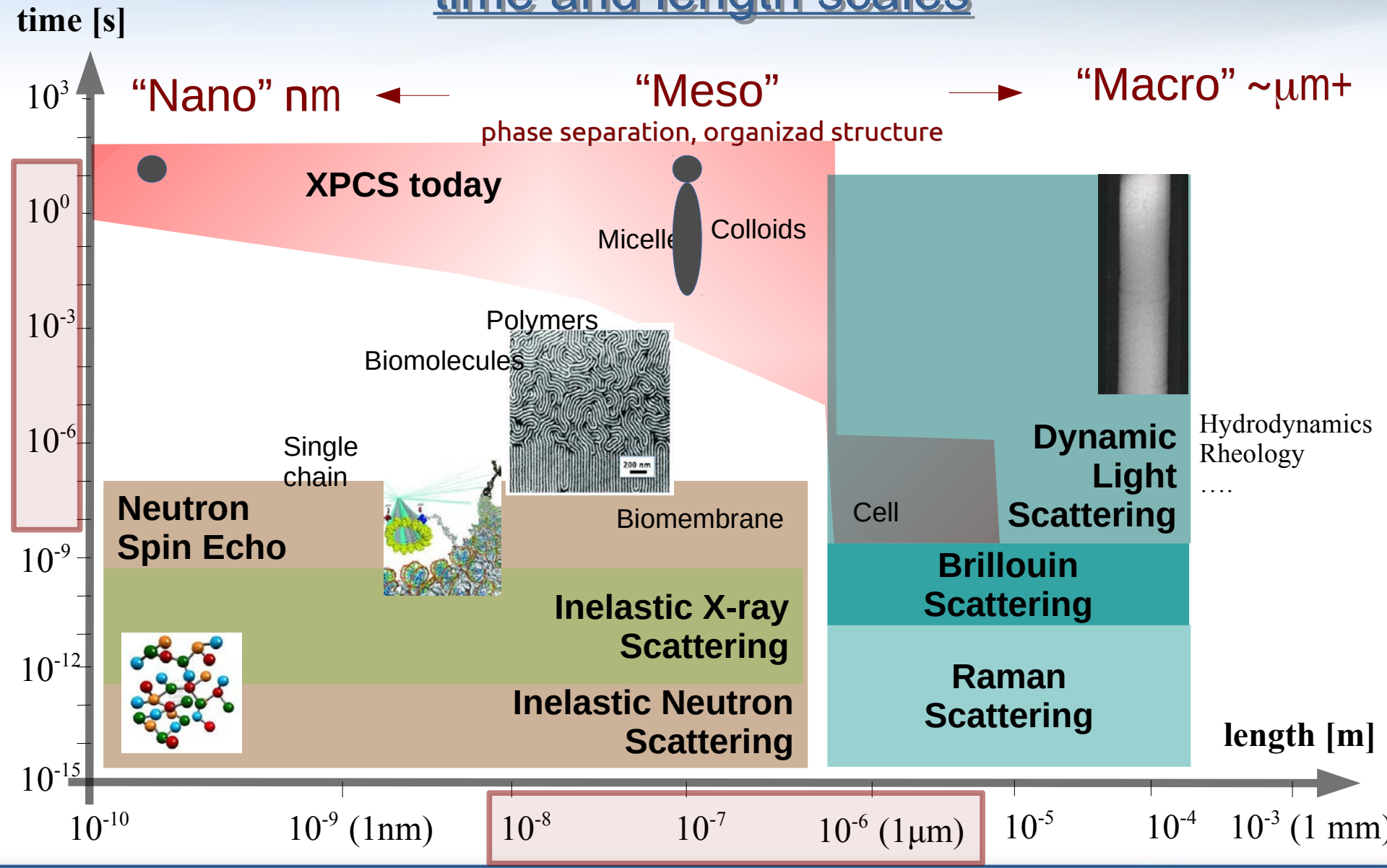
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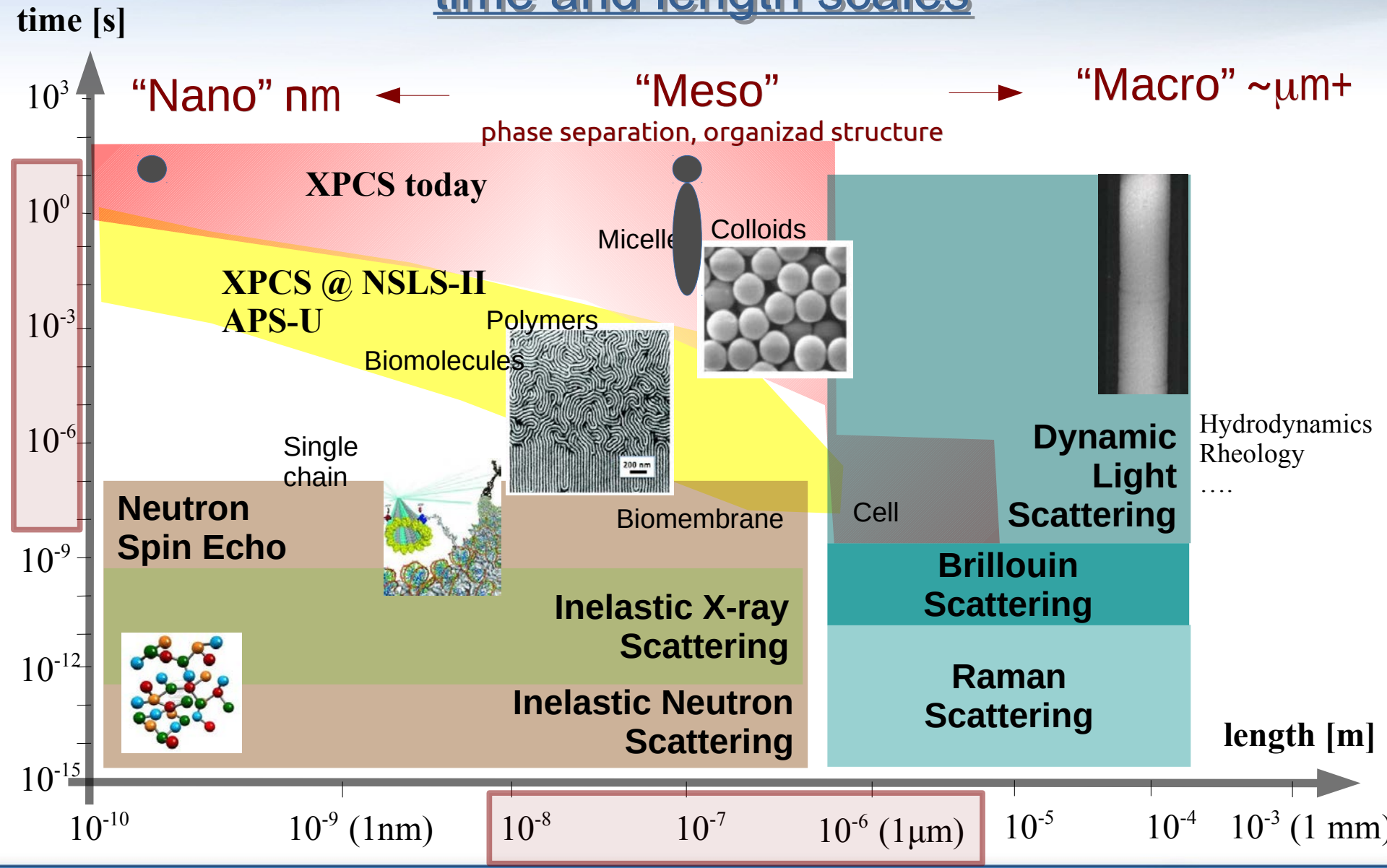
# Dynamics of Materials (soft- and bio-): time and length scales



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# Dynamics of Materials (soft- and bio-): time and length scales



# Colloids

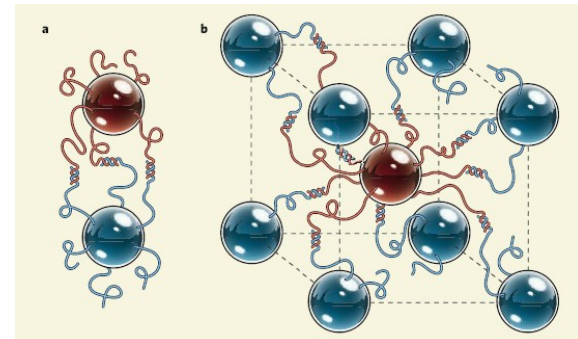
- Colloids are ubiquitous:

- Particles (1-1000 nm) of dispersed phase in dispersion medium



- Phase behavior;  
The “magic” of self-assembly ...

- Opals are dried “polycrystalline” colloids” patchy colloids” can be elementary blocks for programmable self-assembly of “colloidal materials”  
(O. Gang, BNL & Columbia)
- etc



# Brownian Motion; Fluctuations

How far does a particle move in a time  $t$  due to Brownian motion (diffusion):

This is given by the *mean-squared displacement*  $\langle r^2 \rangle$  and it varies linearly with time

$$\langle r^2 \rangle = 6\mathcal{D} t$$

Einstein-Smoluchowski equation

$\mathcal{D}$  = diffusivity of the particles

---

How far the particle moves is dictated, in turn, by its diffusivity  $\mathcal{D}$ :

$$\mathcal{D} = \frac{k_B T}{6\pi\eta_s R_h}$$

Valid for monodisperse  
spherical particles

Stokes-Einstein equation

- As absolute temperature  $T$  increases, diffusivity increases
- As solvent viscosity  $\eta_s$  increases, diffusivity decreases
- As particle size (hydrodynamic radius,  $R_h$ ) increases, diffusivity decreases

# Intermediate Scattering Function

The intermediate scattering function  $f(Q, \tau)$  is related to the static structure factor  $S(Q)$  of the sample and

$$f(Q, \tau) = \frac{1}{S(Q)} \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \langle \exp(i\mathbf{Q} \cdot [\mathbf{r}_i(0) - \mathbf{r}_j(\tau)]) \rangle \quad (18)$$

for scattering from  $N$  identical particles in the illuminated volume. The simplest possible dynamics is Brownian motion (Stokes-Einstein free diffusion) of such  $N$  particles [16, 42]. In the absence of any interactions between particles  $S(Q) = 1$  and all the cross terms ( $i \neq j$ ) in Eq. 18 average out to zero. The mean-square value of the displacement of a free Brownian particle is

$$\langle [\mathbf{r}_i(0) - \mathbf{r}_j(\tau)]^2 \rangle = 6D_0\tau \quad (19)$$

where  $D_0$  denotes the free diffusion coefficient of a particle with radius  $R_p$  in a medium with viscosity  $\eta$  and

$$D_0 = \frac{k_B T}{6\pi\eta R_p}. \quad (20)$$

In this case one finds

$$f(Q, \tau) = \exp(-D_0 Q^2 \tau). \quad (21)$$

The *intermediate scattering function* is typically calculated in numerical simulations and can be measured by experimental techniques such as *Dynamics Light Scattering (DLS)*, *X-ray Photon Correlation Spectroscopy (XPCS)*, *Neutron Spin Echo (NSE)*

# Characteristic timescales

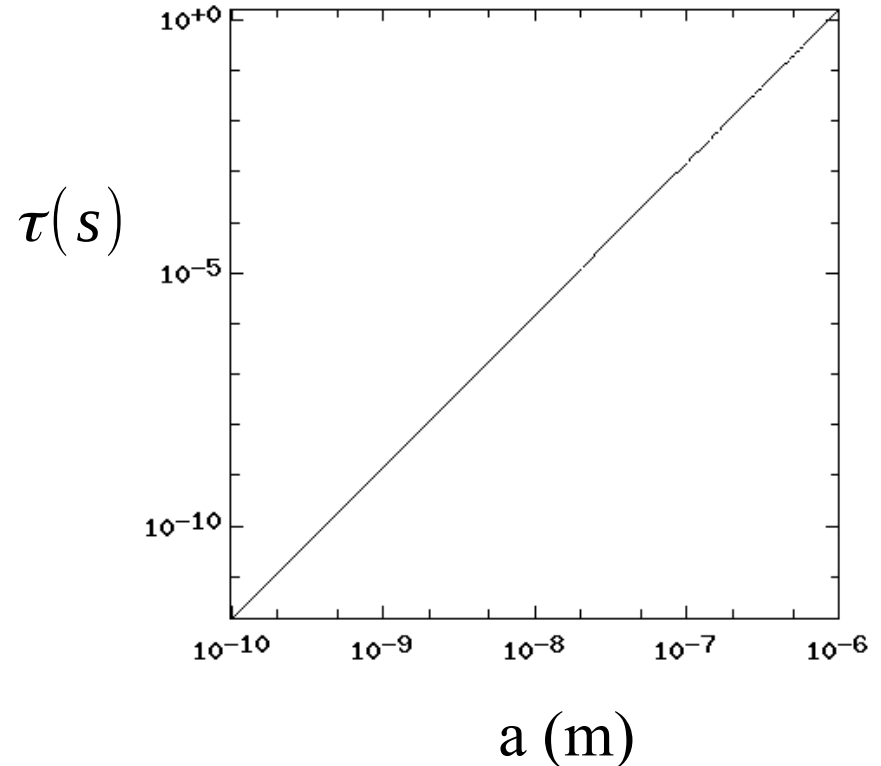
How much time does it take for a particle to travel (diffuse) a distance equal to its diameter  $a$ ?

This time, from Einstein-Smoluchowski eq. is

$$\tau = \frac{a^2}{6D}$$

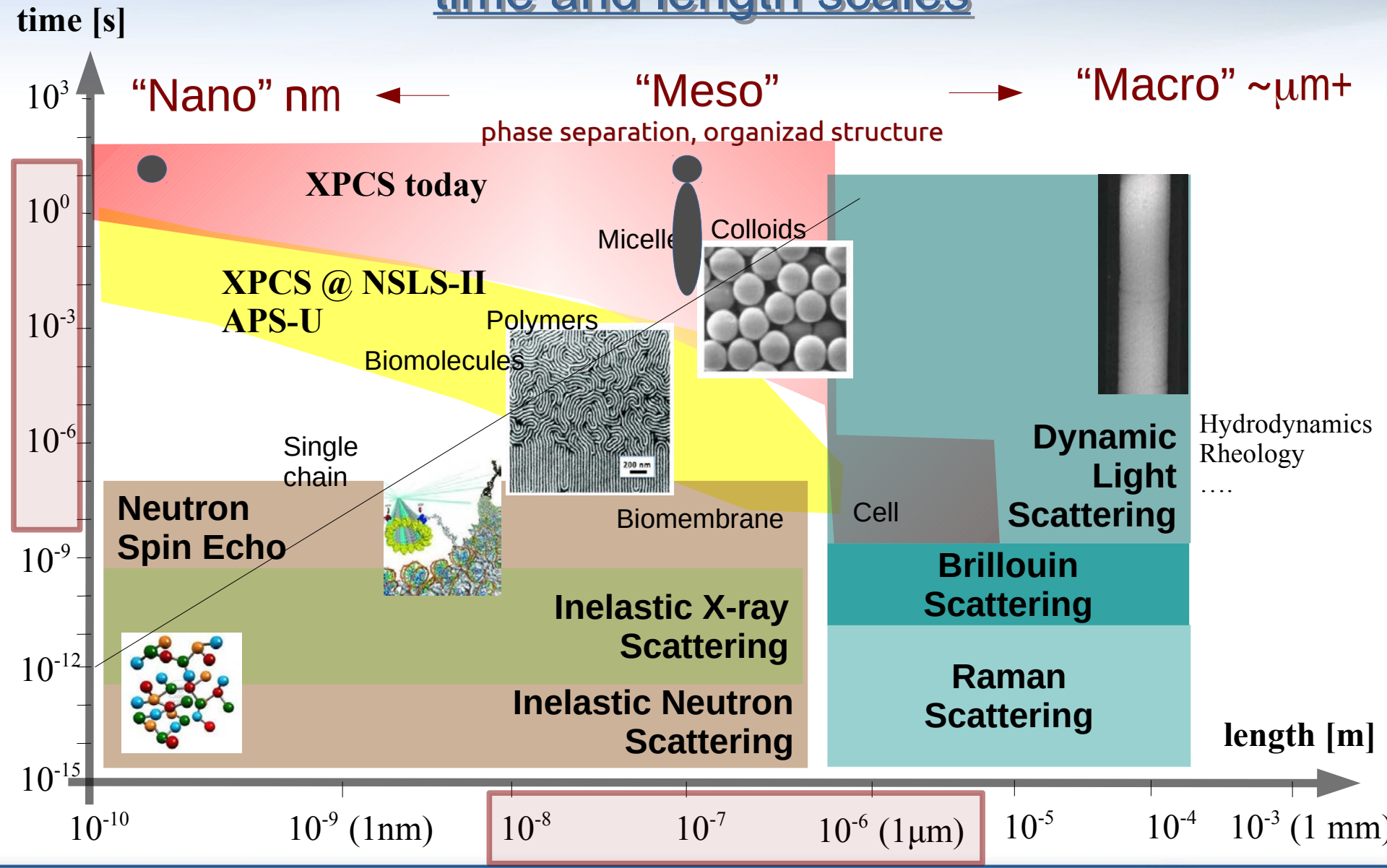
where (Stokes-Einstein)

$$D = \frac{k_B T}{6\pi\eta a}$$





# Dynamics of Materials (soft- and bio-): time and length scales



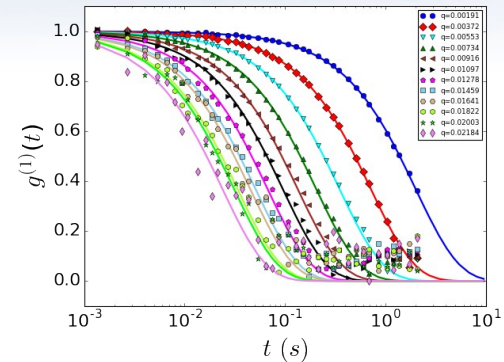
# Density-density correlation functions

## Dynamic Structure Factor & ISF

- Intermediate Scattering Function:

$$f(\vec{q}, t) = \frac{1}{S(\vec{q})} \int G(\vec{r}, t) \exp[-i\vec{q}\vec{r}] d\vec{r}$$

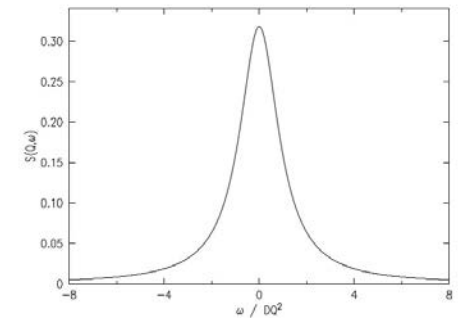
can be measured by experimental techniques such as Neutron Spin Echo (NSE), X-ray Photon Correlation Spectroscopy (XPCS) and Dynamics Light Scattering (DLS); *ISF is a time-dependent structure factor  $S(q, t)$*   
*For diffusion, the ISF has an exponential shape*



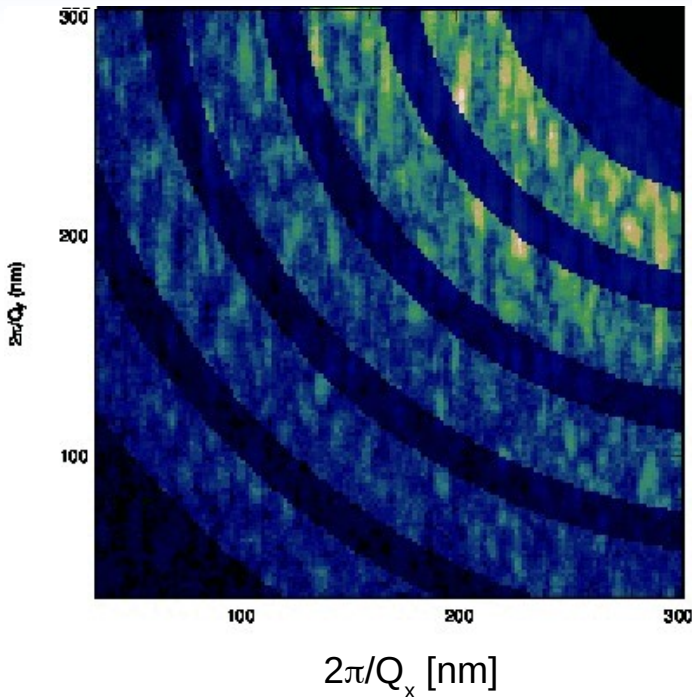
- Dynamic Structure Factor:

$$S(\vec{q}, \omega) = \frac{1}{2\pi} \int F(\vec{q}, t) \exp[i\omega t] dt$$

can be measured by experimental techniques such as Inelastic neutron and X-ray scattering. *The dynamic structure factor is a frequency-dependent structure factor*  
*For diffusion,  $S(q, \omega)$  has a Lorentzian shape.*

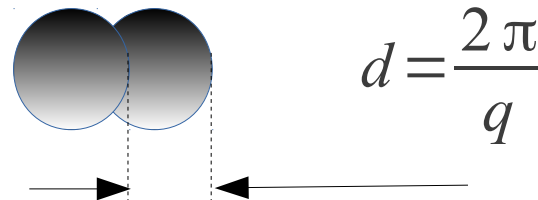


# Colloidal Dynamics with XPCS

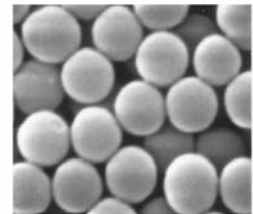


$$q = \frac{4\pi}{\lambda} \sin\left(\frac{2\theta}{2}\right)$$

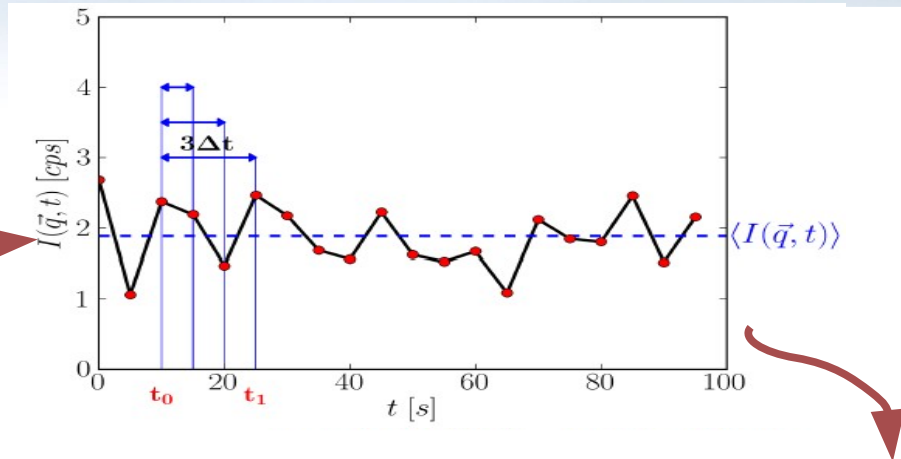
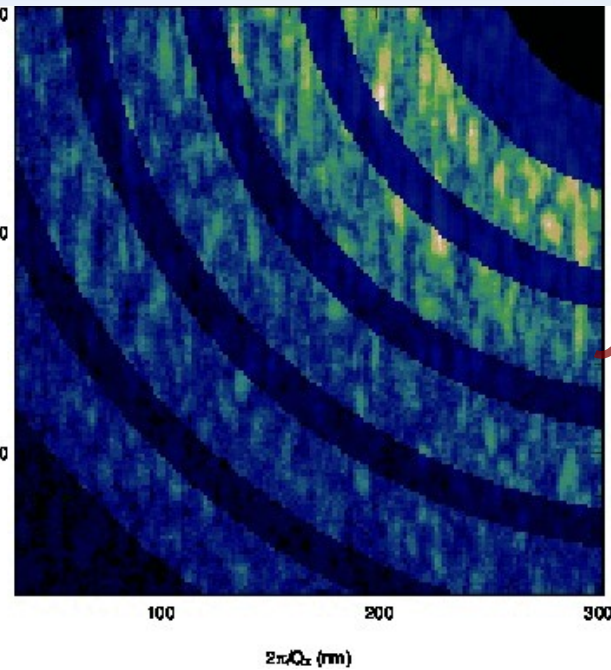
- Measures time scale associated with displacement of colloids



- i.e. measures dynamic structure factor  $S(q,t)$
- By averaging over  $\sim 10^{11}$  particles
- For different  $q$  values

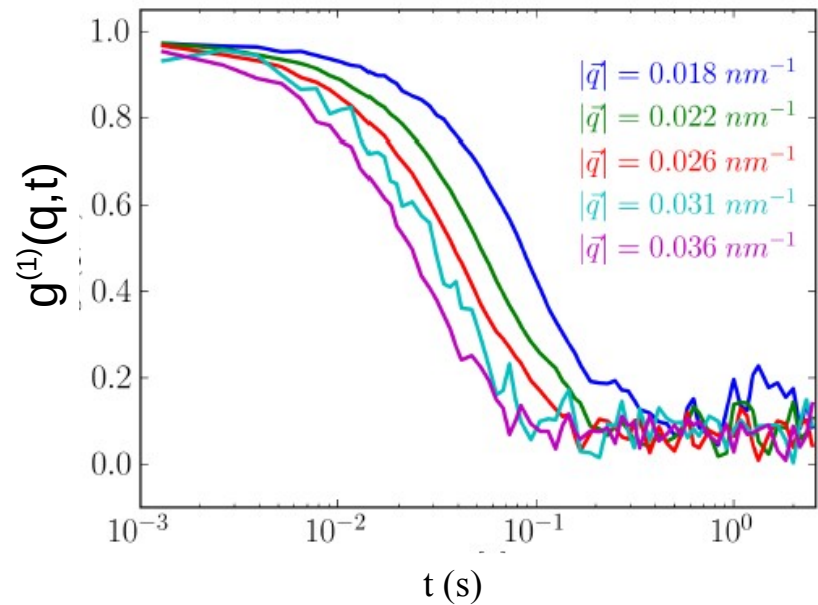


# Colloidal Dynamics with XPCS



$$g^{(2)}(q, t) = \frac{\langle I(q, t) I(q, t + \delta t) \rangle}{\langle I(q) \rangle^2}$$

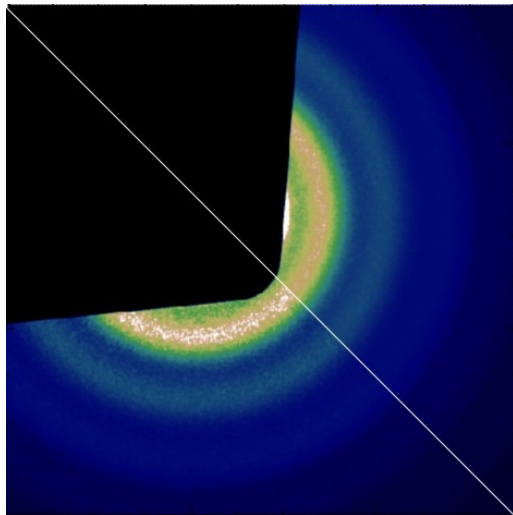
$$g^{(2)}(q, t) = 1 + \beta(q) [g^{(1)}(q, t)]^2$$



# Speckle & Intensity Autocorrelation Functions

- Coherence & Intensity autocorrelation functions of the speckle patterns measure the intermediate scattering function (a.k.a. dynamics)

$$\langle I(q, t) I(q, t + \tau) \rangle = \langle I(q) \rangle^2 [1 + \beta(q) (\dots) |f(q, \tau)|^2]$$



“Incoherent” scattering ( from any SAXS experiment/instrument)

Coherent Scattering / Speckle

# Diffusive Dynamics; Correlation Functions

- Intensity autocorrelation functions, ISF and the Siegert relationship:

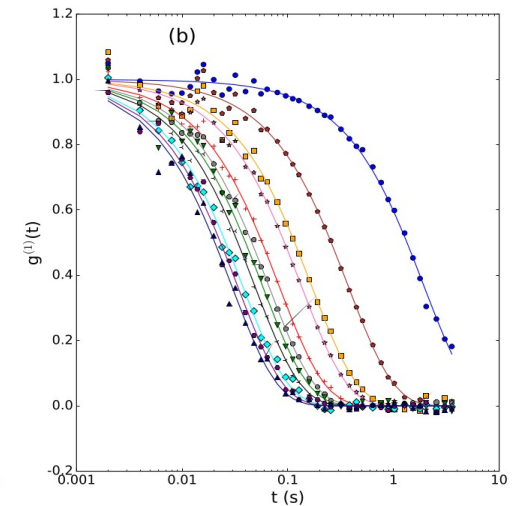
$$g^{(2)}(q, t) = \frac{\langle I(q, t) I(q, 0) \rangle}{\langle I(q) \rangle^2} = 1 + \beta(q) |f(q, t)|^2$$

- Intermediate Scattering Function

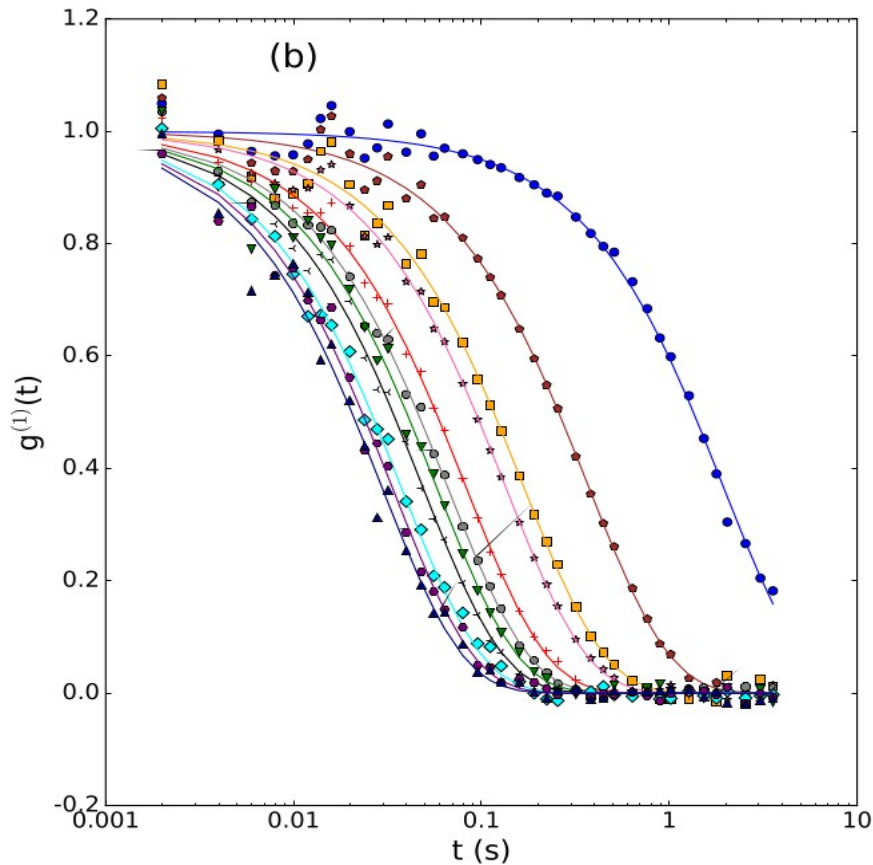
$$g^{(1)}(q, t) = f(q, t) = \exp(-D_0 q^2 t)$$

- Mean square displacement

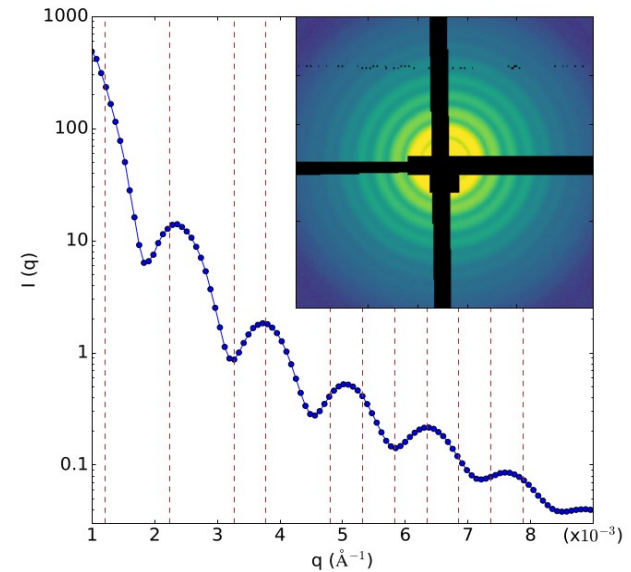
$$\langle [r_i(0) - r_j(t)]^2 \rangle = 6 D_0 t \quad D_0 = \frac{k_B T}{6 \pi \eta a}$$



# Diffusive Dynamics; Correlation Functions



- $q = 0.0012$
- ◆  $q = 0.0022$
- $q = 0.0033$
- ☆  $q = 0.0038$
- +  $q = 0.0048$
- $q = 0.0053$
- ▼  $q = 0.0058$
- ⋈  $q = 0.0063$
- ◆  $q = 0.0069$
- ◆  $q = 0.0074$
- ▲  $q = 0.0079$

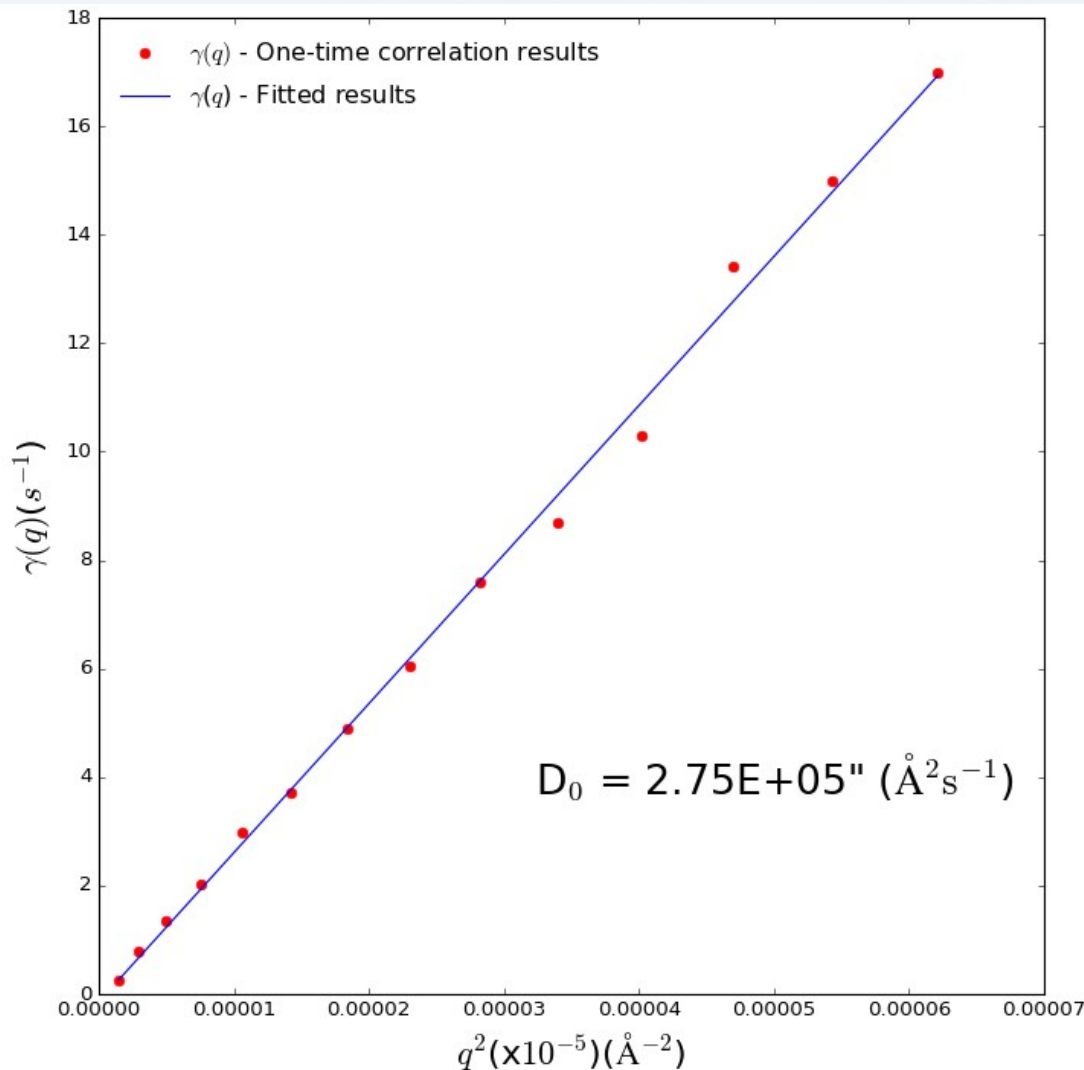


Sample: 500 nm Silica spheres  
suspended in a water/glycerol  
mixture

Length scales:  $d = \frac{2\pi}{q}$

Length scales:  $\sim 80 \text{ nm} - 500 \text{ nm}$

# Colloidal Dynamics with XPCS



Here 500 nm Silica spheres suspended in a water/glycerol mixture

ISF:

$$g^{(1)}(q, t) \propto \exp[-Dq^2 t]$$

Relaxation rate:

$$\gamma = Dq^2$$

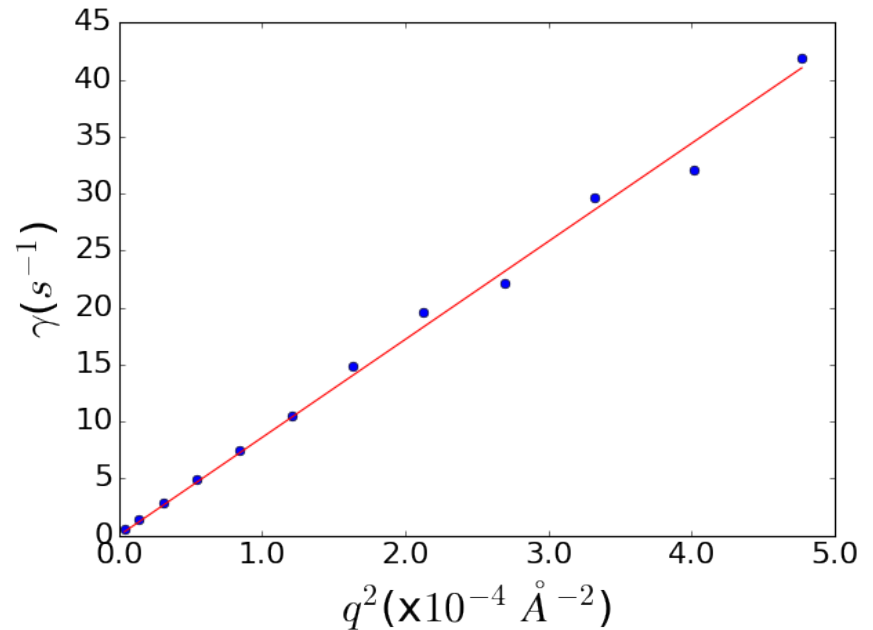
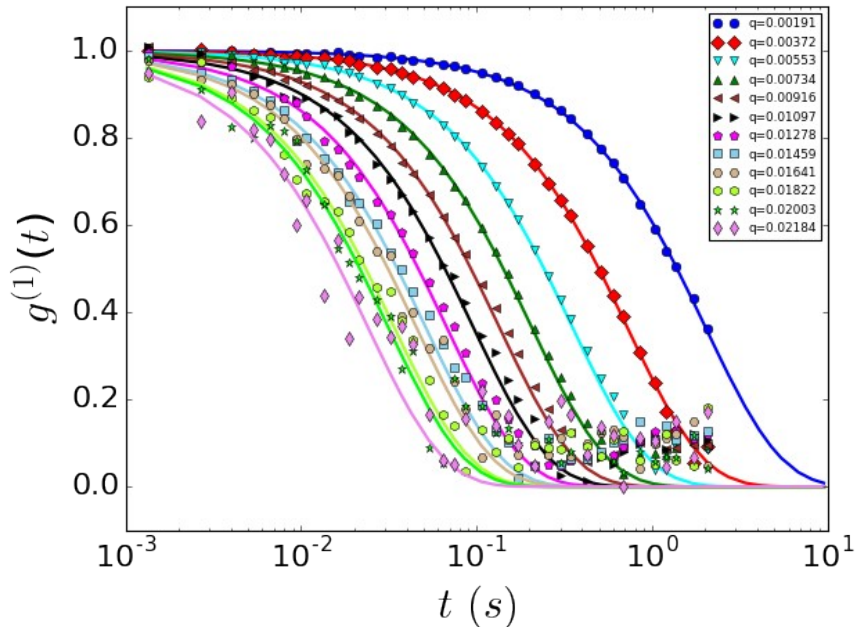
CHX Analysis Pipeline!



# Colloidal Dynamics with XPCS

ISF:  $g^{(1)}(q, t) \propto \exp[-Dq^2 t]$

Relaxation rate:  $\gamma = Dq^2$



Sample: 15 nm Au nanoparticles stabilized (citric acid) and suspended in a polymer liquid 2.7K PEG

# Colloidal Dynamics with XPCS

- Width function analysis

(Martinez, Van Meegen *et al.* JCP 2011)

$$\text{ISF: } g^{(1)}(q, t) \propto \exp[-Dq^2 t]$$

$$\text{MSD: } MSD \propto Dt$$

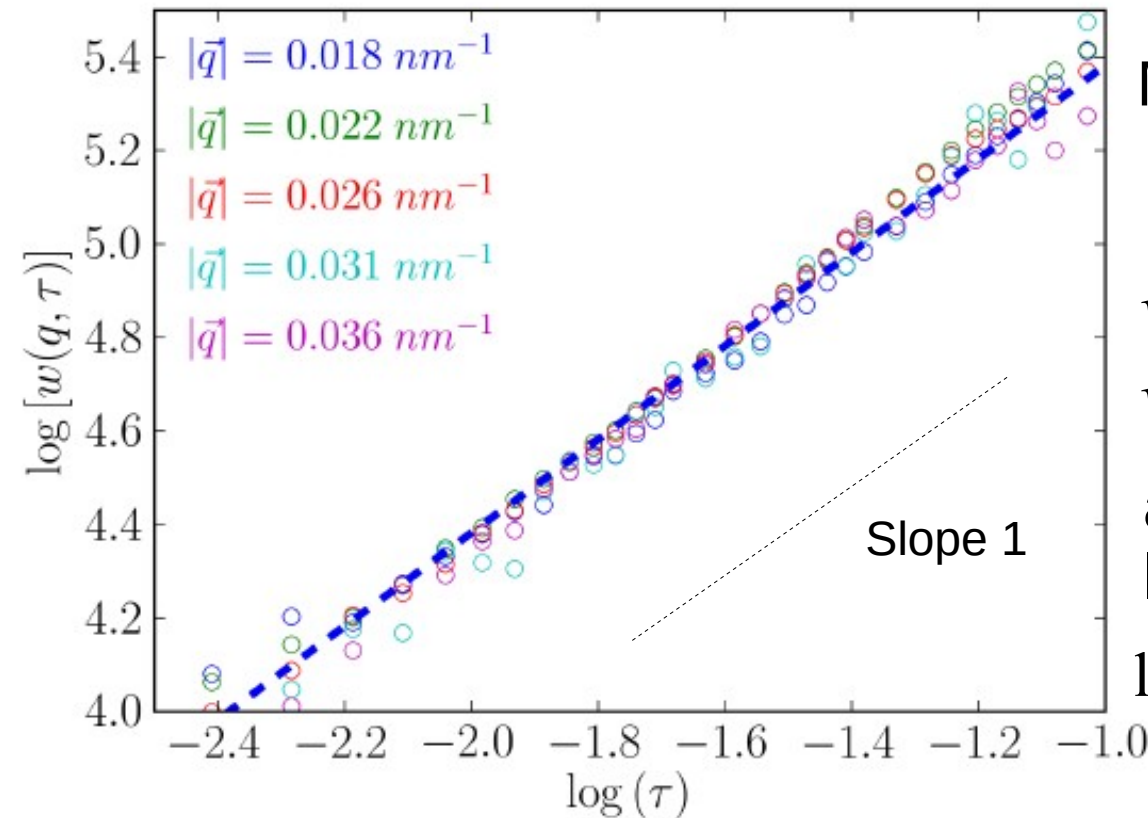
hence:

$$w(q, t) = -\log[g^{(1)}(q, t)/q^2]$$

$$w(q, t) = Dt$$

a.k.a. “width function” is like MSD and

$$\log[w(q, t)] = \log[D] + \log[t]$$

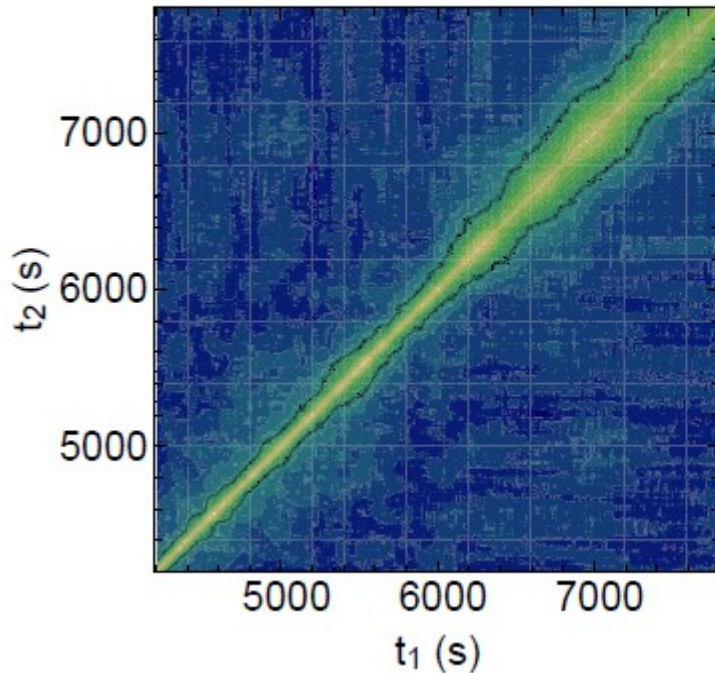


Sample here: PMMA 103 nm particles in cis-decalin

# Two-time analysis

Non-equilibrium dynamics in colloidal depletion gels (colloid/polymer mixtures):

Two-time correlation functions:  $C(Q, t_1, t_2) = \frac{\langle I(Q, t_1) I(Q, t_2) \rangle_{pix}}{\langle I(Q, t_1) \rangle_{pix} \langle I(Q, t_2) \rangle_{pix}}$

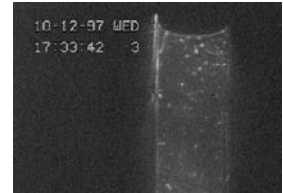


average time (“age”):

$$t_a = \frac{t_1 + t_2}{2}$$

time difference:

$$t = \delta t = |t_1 - t_2|$$

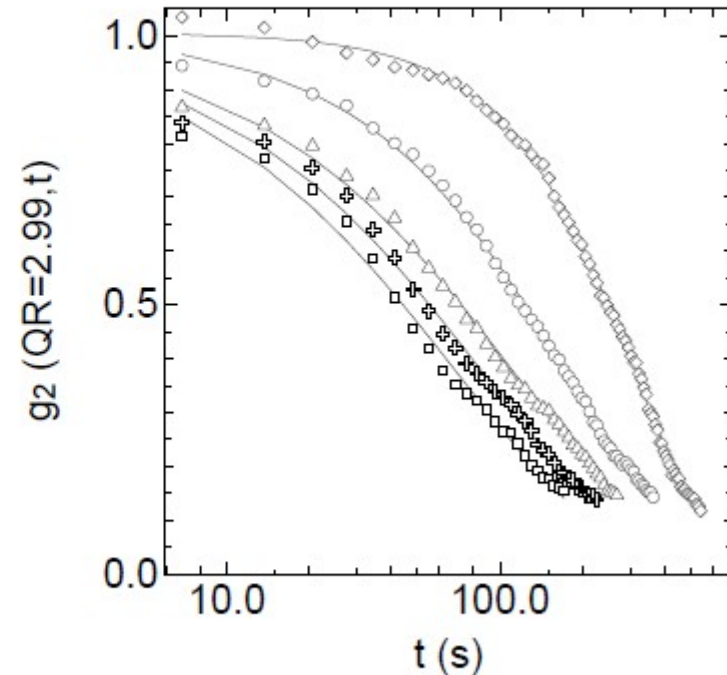
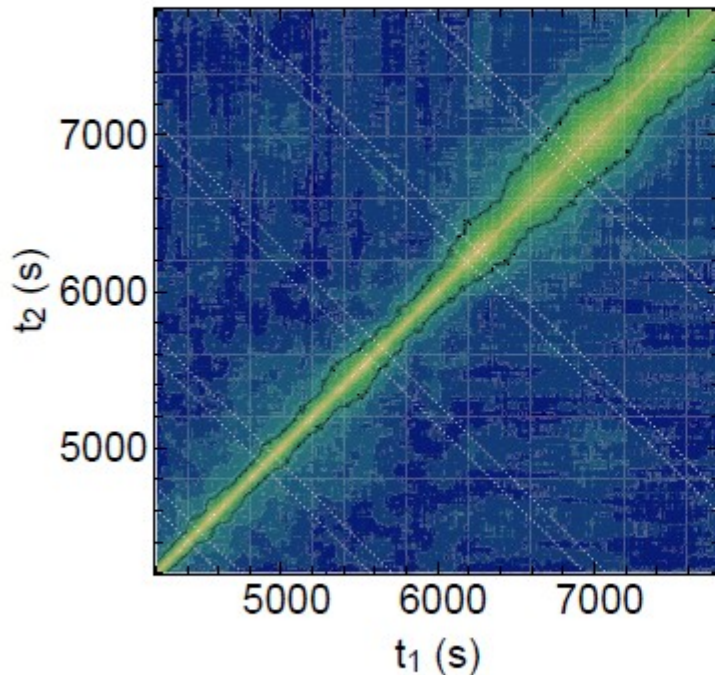


\* M.Sutton et al., Optics Express 11, 2268 (2003).

AF et al., Phys. Rev. E, **76**, 010401(R) (2007)

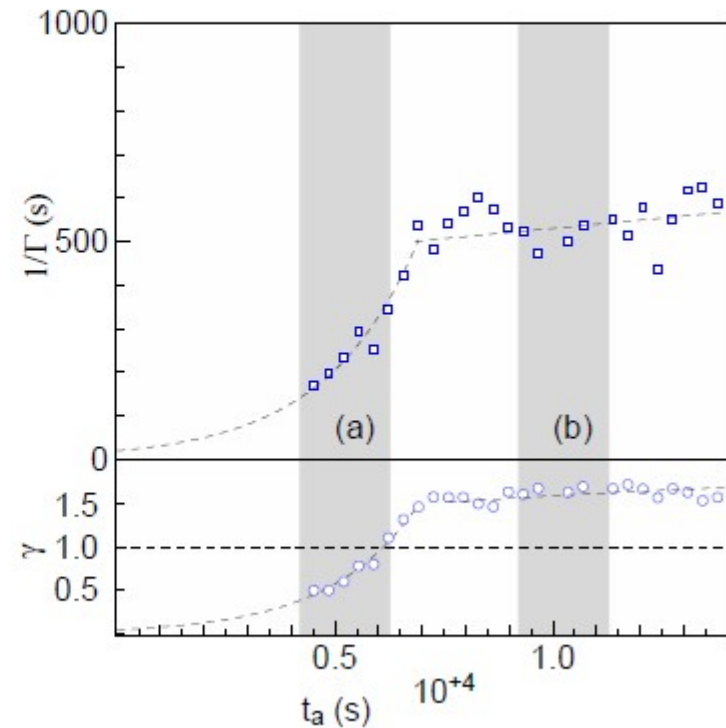
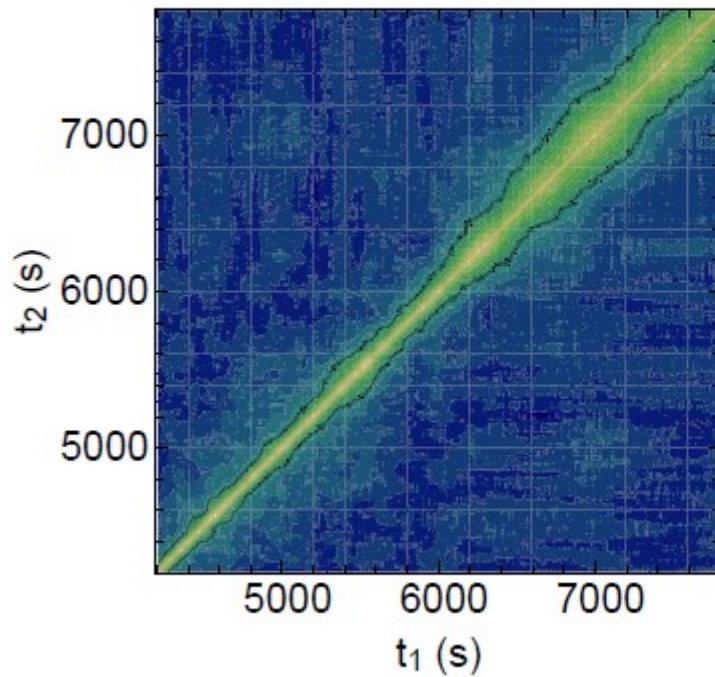
# Two-time analysis

Two-time correlation functions:  $C(Q, t_1, t_2) = \frac{\langle I(Q, t_1) I(Q, t_2) \rangle_{pix}}{\langle I(Q, t_1) \rangle_{pix} \langle I(Q, t_2) \rangle_{pix}}$



# Two-time analysis

Two-time analysis:  $g_2(Q, t_a, t) = \beta \exp(-(\Gamma t)^\gamma) + g_\infty$



# A “User Guide” to XPCS

- CHX optimized for Coherent X-ray Diffraction - XPCS, (GI-)SAXS/WAXS, CDI

Unprecedented q-range available in-situ from Angstroms to Microns

- Source: IVU 20 (low  $\beta$ ) - highest brightness  $E=6-15$  keV

- Beamline Optics: optimized for high stability & wavefront preservation

- COHERENT FLUX:  
 $\approx 10^{11}$  ph/sec ( $\Delta\lambda/\lambda=10^{-4}$ )  
 $\approx 10^{12}$  ph/sec ( $\Delta\lambda/\lambda=10^{-3}$ )

- BEAM SIZE :  
 $\approx 10$   $\mu\text{m}$  (SAXS)  
 $\approx 1$   $\mu\text{m}$  (WAXS)

## DETECTORS

### 1. Diagnostics

- Fluorescent Screens; Pin diodes, Monitor counter; beam imaging; BPM

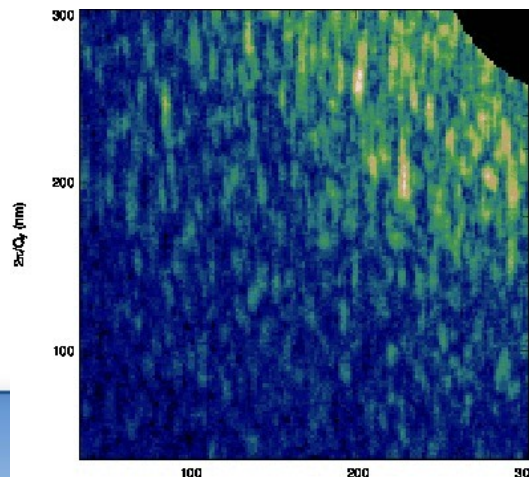
### 2. EIGER (Dectris)

best in class area detectors 3kHz (up to 15 kHz), 75  $\mu\text{m}$  pixels

- Eiger 1M for c - WAXS
- Eiger 4M for c - SAXS

### 3. Point Detectors (FMB Oxford)

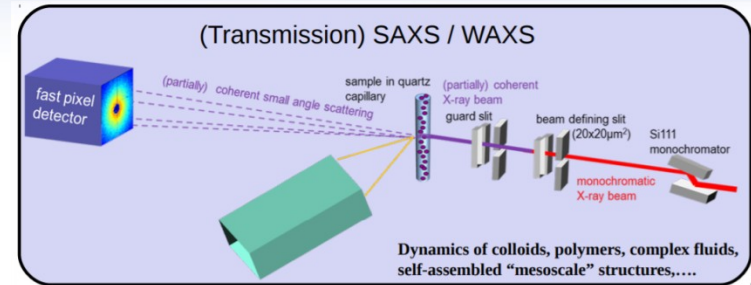
- Scintillator detector systems;
- Avalanche Photodiode (APD)



# XPCS & different Scattering Geometries

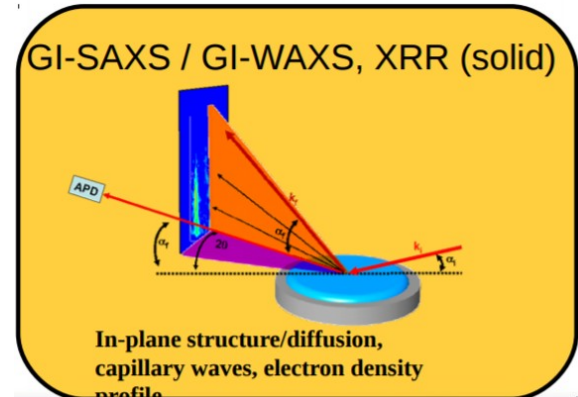
- **SAXS**

e.g. for studies of the interplay between nanoscale and mesoscale structure, dynamics and macroscopic properties.



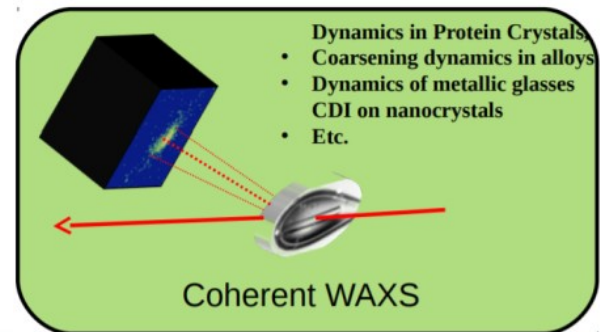
- **GI-SAXS**

e.g. for studies of dynamical phenomena at surfaces and interfaces during thin film growth



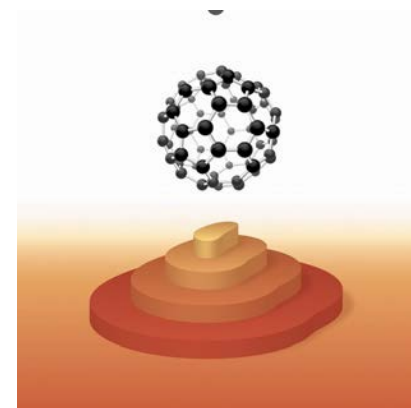
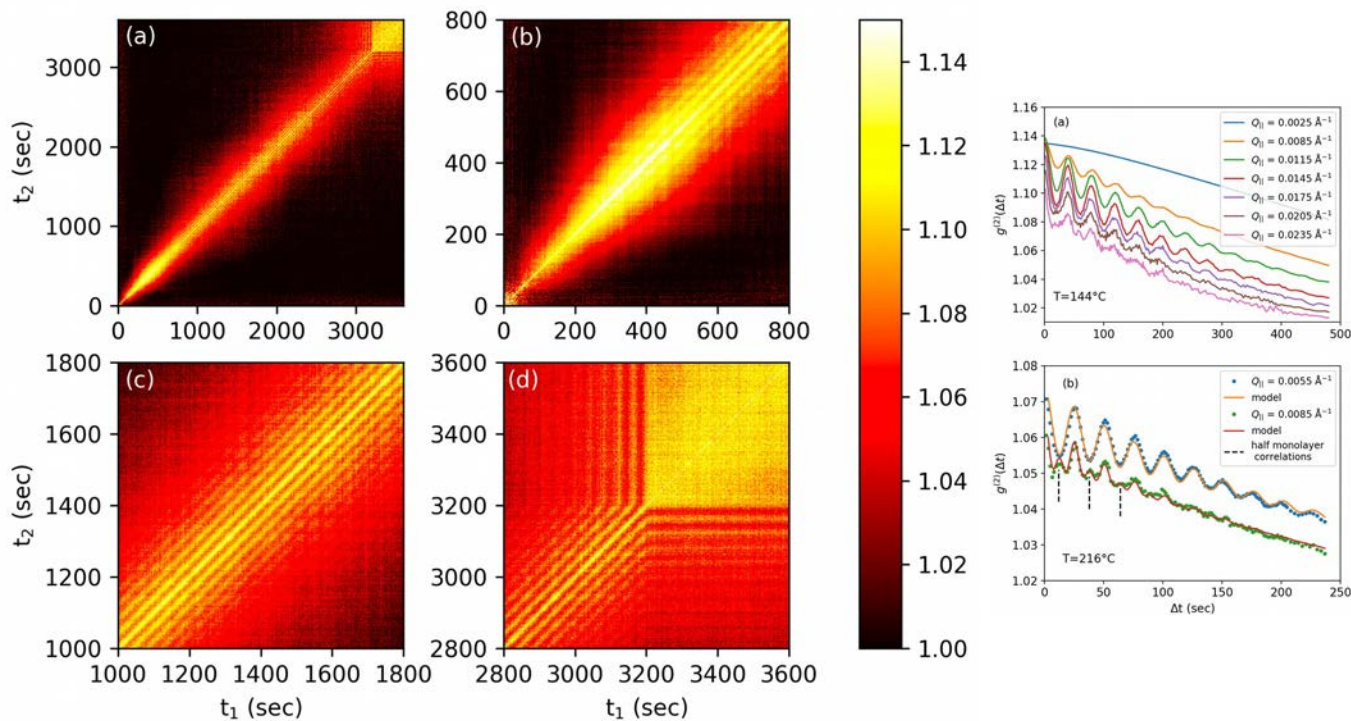
- **WAXS**

e.g. for studies of dynamics of ferroelectric domains in ferroelectric periodic heterostructures



# Example: GI-XPCS during in-situ growth

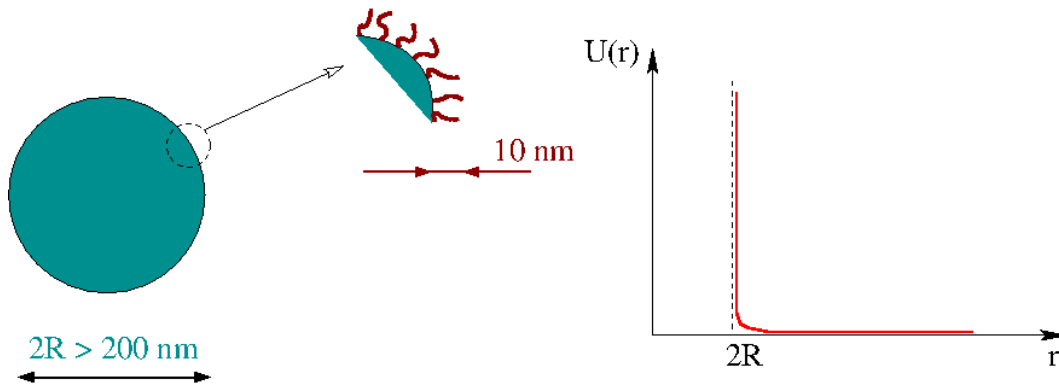
- The research is of a major fundamental and practical importance, providing a better understanding of the properties of artificially grown thin films.
- GI-XPCS -> understanding aspects such as step flow and other out-of-equilibrium fluctuations which are impossible to access with other techniques



R. Headrick *et al.*, Nature Comm. 2019



# A more detailed science example: high density hard-sphere (colloidal) suspensions

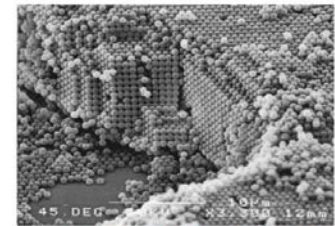
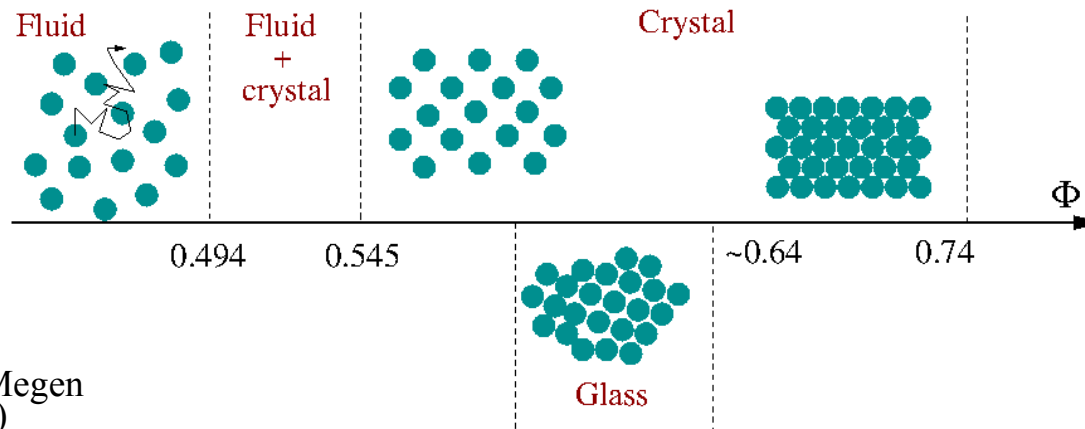


## Hard-sphere colloids:

- Spherical PolyMethylMethacrylate (PMMA) particles coated with 12 hydroxystearic acid in cis-decalin (A. Schofield, Edinburgh)
- Entropic forces between polymer coating layers → infinite “hard-sphere-like” repulsions

- The phase behavior depends on the *particle volume fraction*  $\Phi$

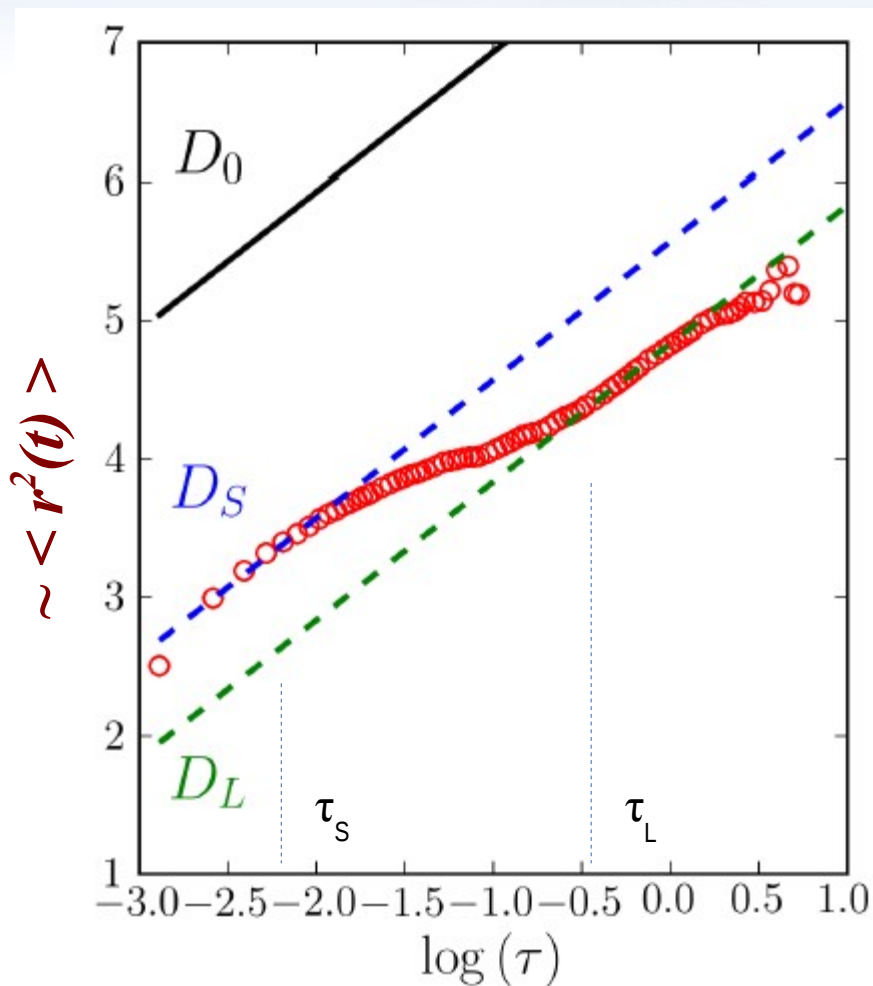
$$\Phi = \frac{N V_{colloid}}{V_{total}}$$



(b) R.M. Amos et al., PRE 61, 2929 (2000)

P.N. Pusey & W. Van Meegen  
*Nature* **320**, 340 (1986)

# Dynamics in high density hard-sphere suspensions



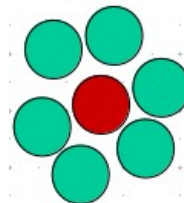
P. Kwasniewski, PhD Thesis 2012

## Short-time diffusion $D_s$ ( $t < \tau_s$ )

Motion of particles inside of “cages”  
created by other particles

Slowed down (compared to  $D_0$ ) by  
*hydrodynamic interactions*

D. Orsi, AF et al. *Phys. Rev. E* 2012



## Long-time diffusion $D_L$ ( $t > \tau_L$ )

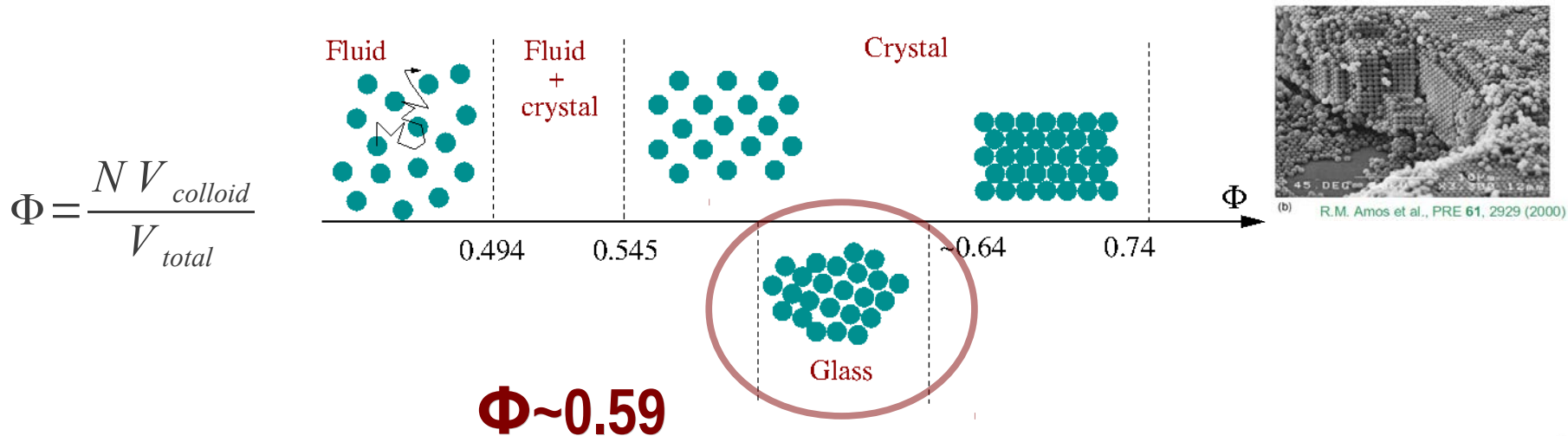
Structural rearrangements i.e.  
“Rearrangements of cages”

Slowed down (compared to  $D_s$ ) by  
*direct interactions*

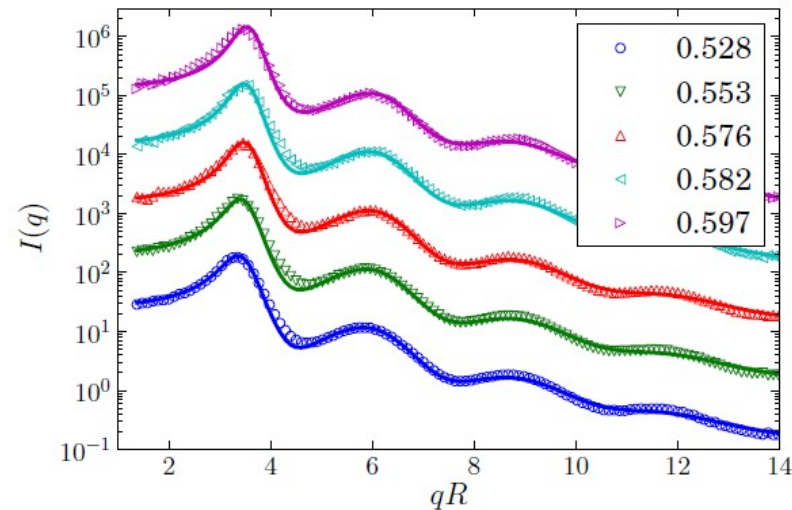
P. Kwasniewski, AF, A. Madsen, *Soft Matter*, 2014, **10**, 8698-8704

# The Colloidal Glass Transition

- What happens here?

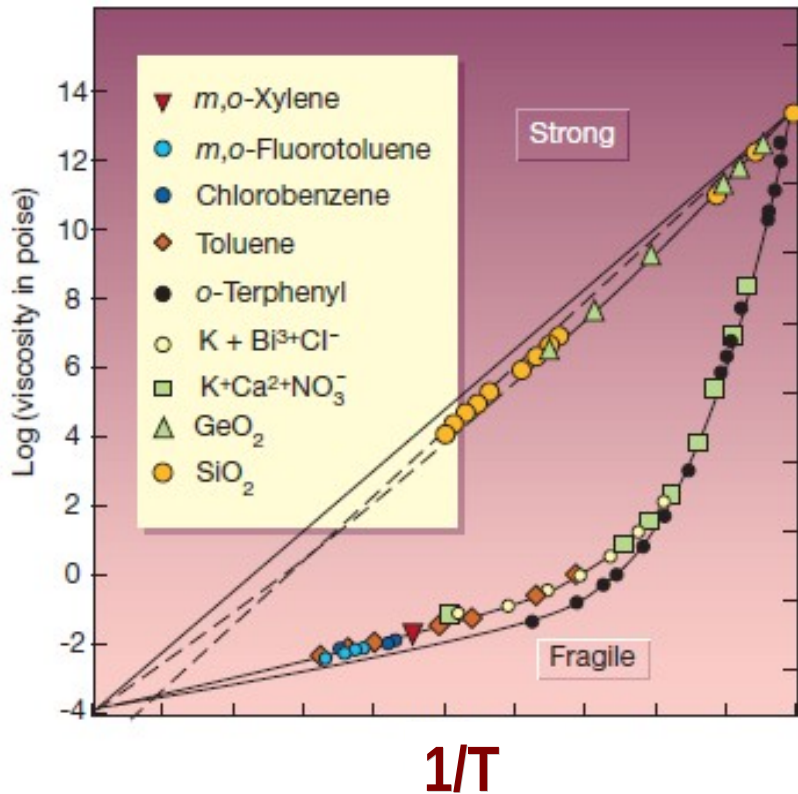


- From SAXS / static scattering: pretty much *nothing* ...

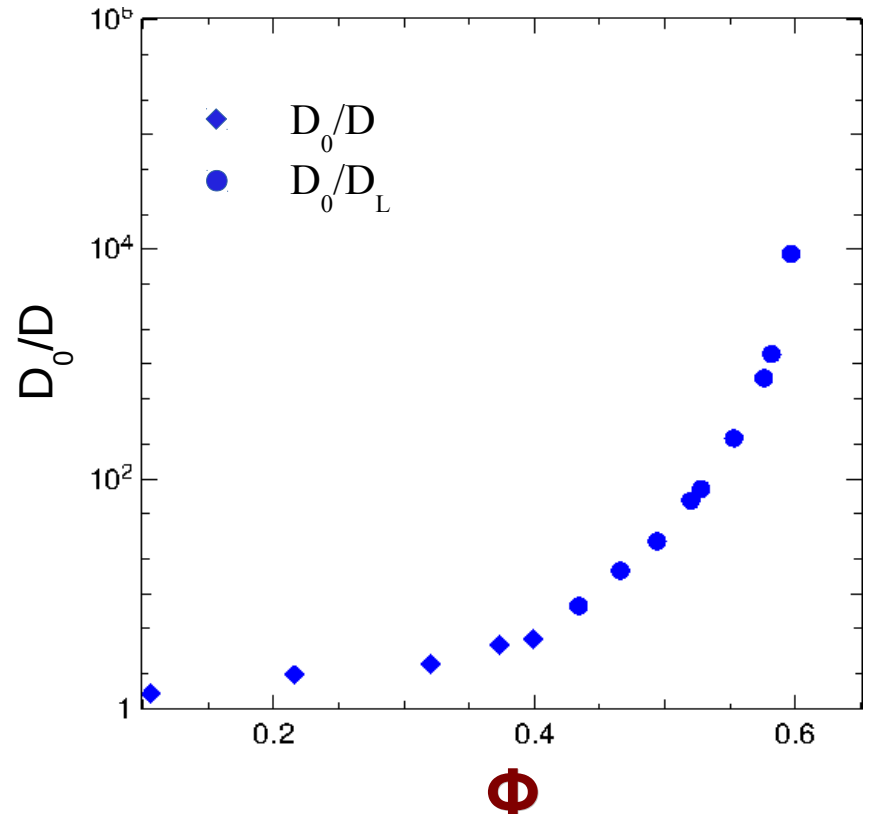


# Supercooled Liquids vs. Hard-Sphere Colloids

- In addition to being interesting/useful in their own right, colloids are an excellent model system for supercooled liquids and molecular glassformers



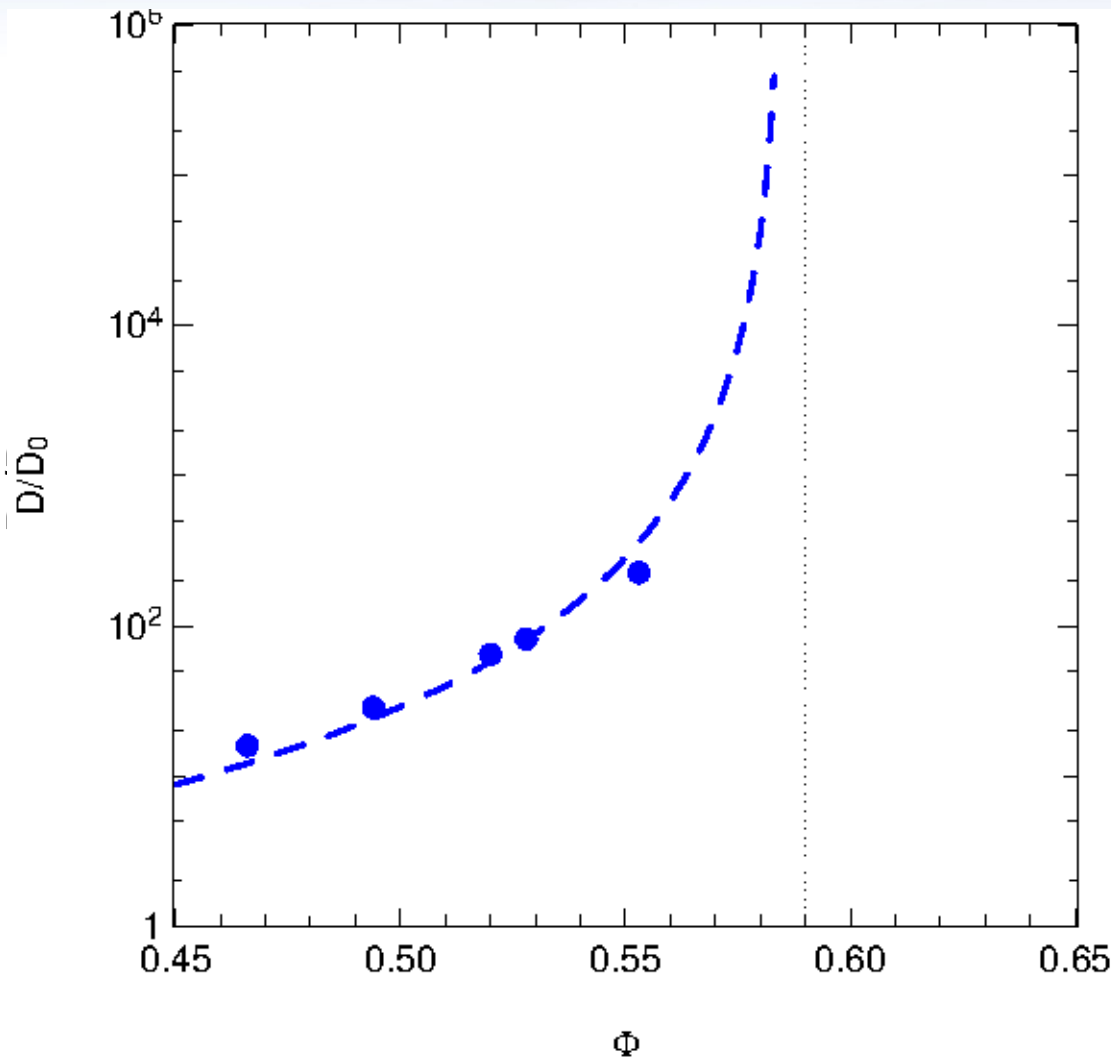
Denenedetti, Stillinger, *Nature* 2001



D. Orsi, AF et al. *Phys. Rev. E* 2012

P. Kwasniewski, AF, A. Madsen, *Soft Matter* 2014  
 $\eta/\eta_0 \rightarrow D_0/D_L$  (Segre et al., *Phys. Rev. Lett* 2001)

# Structural Relaxations near the Hard-Sphere Glass Transition

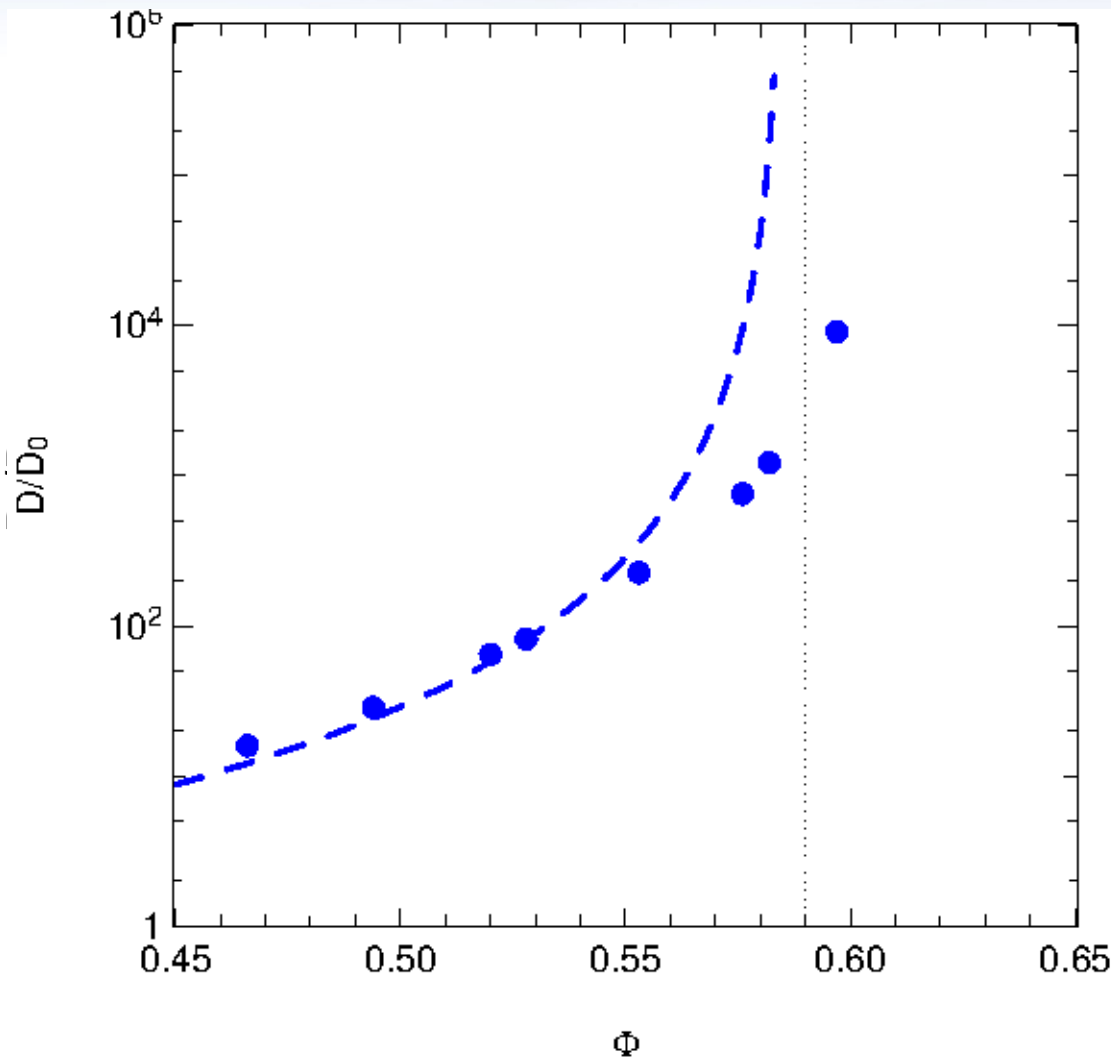


## Structural relaxations:

- Structural relaxations slow-down with increasing  $\Phi$
- And are expected to *diverge* at the colloidal glass transition concentration  $\Phi_g$  - "Mode Coupling Theory"-(MCT)
- $D_0/D_L \rightarrow \infty$  at  $\Phi_g \sim 0.59$

$$\frac{D_0}{D_L(q_m)} \propto \left| \frac{\Phi_g - \Phi}{\Phi_g} \right|^{-\gamma}$$

# Structural Relaxations near the Hard-Sphere Glass Transition



## Structural relaxations:

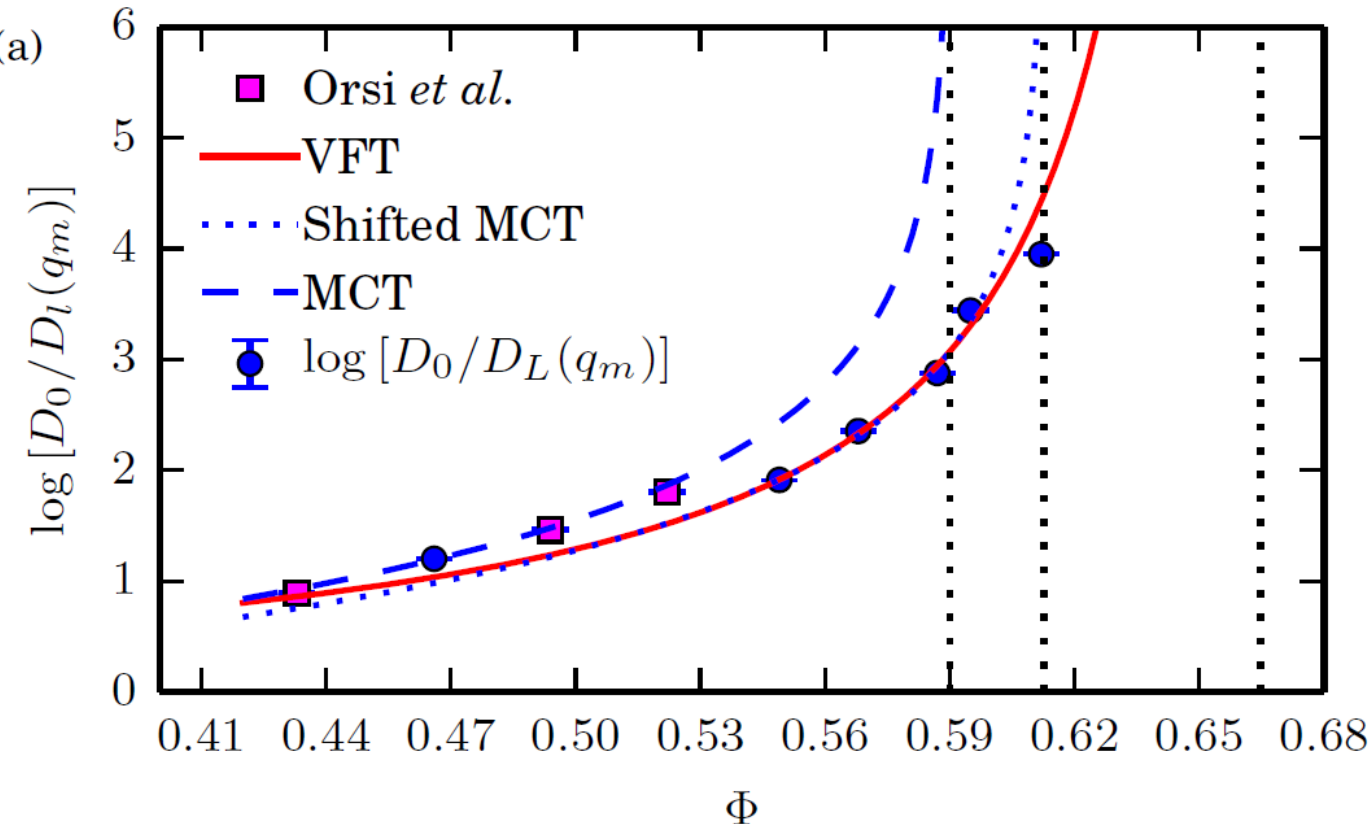
- Structural relaxations slow-down with increasing  $\Phi$
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$$\frac{D_0}{D_L(q_m)} \propto \left| \frac{\Phi_g - \Phi}{\Phi_g} \right|^{-\gamma}$$

## Not so simple:

- Instead of diverging the relaxations remain finite (but slow!) above  $\Phi_g$

# Structural Relaxations near the Hard-Sphere Glass Transition



MCT:

$$\frac{D_0}{D_l(q_m)} \propto \left| \frac{\Phi_g - \Phi}{\Phi_g} \right|^{-\gamma}$$

$g \sim 2.58$

VFT:

$$\frac{D_0}{D_l(q_m)} = \tau_\infty \exp \left[ \frac{F}{(\Phi_0 - \Phi)^\delta} \right]$$

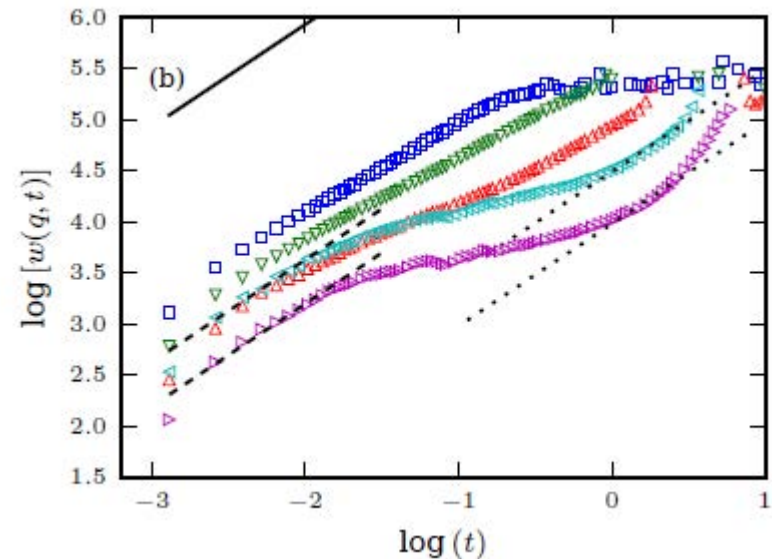
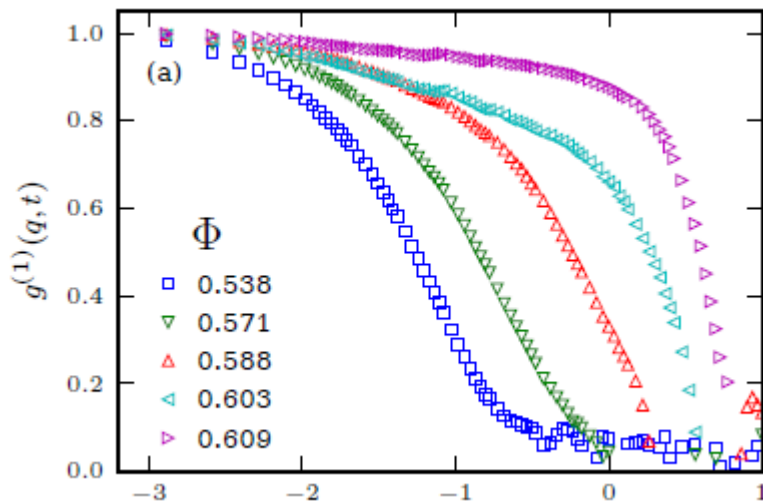
- relaxations follow an unexpected functional (VFT) form suggesting a kinetic arrest near the “random close packing concentration”  $\Phi_{RCP} \sim 0.67$  (~10% polydispersity)
- Suggests connection with **Jamming**

P. Kwasniewski, AF, A. Madsen, *Soft Matter*, 2014, 10, 8698-8704

See also; Brambilla, Cipelletti *et al.*, *Phys. Rev. Lett.* 104, 169602 (2010)

# Anomalous Dynamics near the Hard-Sphere Glass Transition

- Near the colloidal Glass Transition the dynamics becomes anomalous
  - Compressed exponential relaxations
  - Hyperdiffusive dynamics:  $\langle r^2(t) \rangle$  “faster than”  $\sim t$



- Is this behavior a signature of *jamming*?

Universal non-diffusive slow dynamics in aging soft matter  
L.Cipelletti *et al.*, *Faraday Discuss.*, 2003, **123**, 237

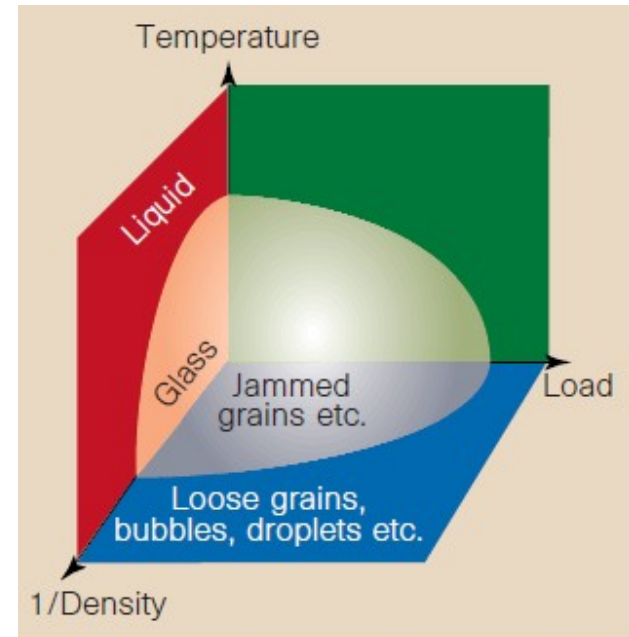


# Jamming?

- Is this behavior a “universal” ?
- Common behavior in seemingly different systems: hyperdiffusive & faster-than-exponential relaxations associated with *Jamming*

L.Cipelletti *et al.*, *Faraday Discuss.*, 2003, **123**, 237

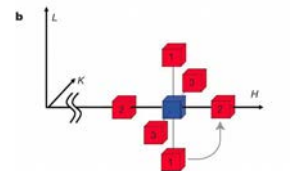
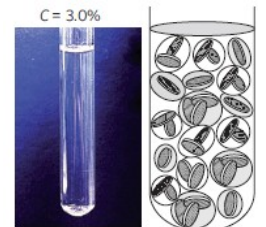
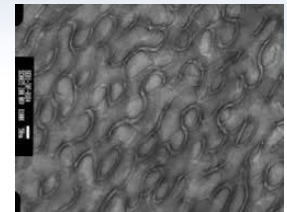
- Jamming – heterogeneities & response to flow/shear



A. Liu *et al.* Nature 1998

# Anomalous Dynamics near the Hard-Sphere Glass Transition

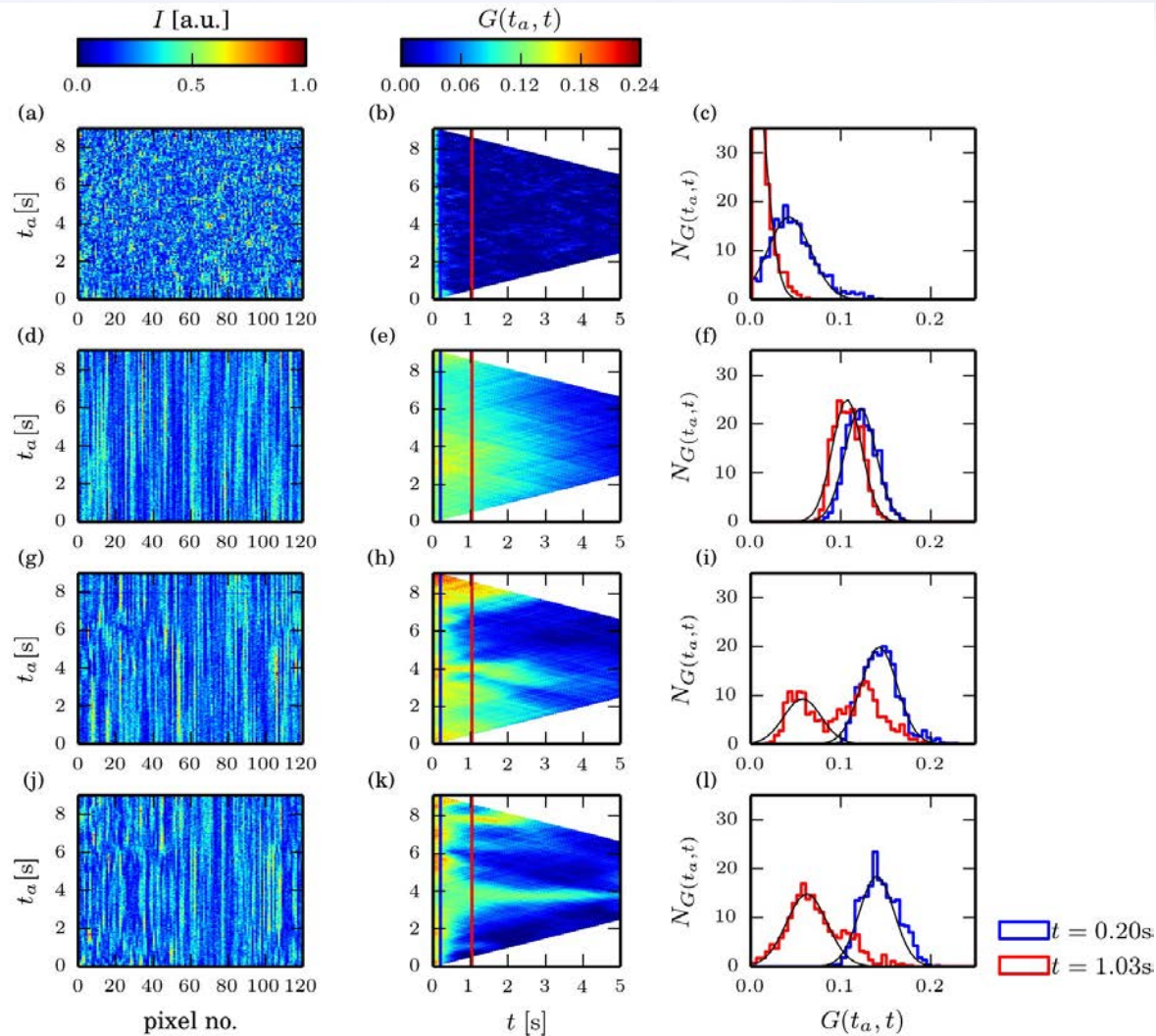
- Polymer-based sponge phases  
P. Falus *et al.* *Phys. Rev. Lett* 2006
- Aging Clay (Laponite) Gels  
B. Bandyopadhyay *et al.*, *Phys. Rev. Lett.* 2004;  
R. Angelini *et al.*, *Soft Matter* 2013
- Antiferromagnetic domain fluctuations (Cr)  
O. Shpyrko *et al.*, *Nature* 2007
- Aging Ferrofluids  
A. Robert *et al.* *Europhys. Lett.* 2007
- Aging colloidal gels (“transient gels”)  
A. Fluerasu *et al.*, *Phys. Rev. E* 2007
- Cross-linked Polymer Gels  
R. Hernandez *et al.*, *J. Chem Phys* 2014  
O. Czakkel, *Europhys. Lett.* 2011, K. Laszlo *et al.*, *Soft Matter* 2010
- Atomic-scale dynamics & aging in metallic glasses  
B. Rutta *et al.*, *Phys. Rev. Lett.* 2012
- Etc. etc. etc. ...



# Dynamical Heterogeneities

$\Phi \sim 0.57$

$\Phi \sim 0.61$



Age  $\sim 30$ min

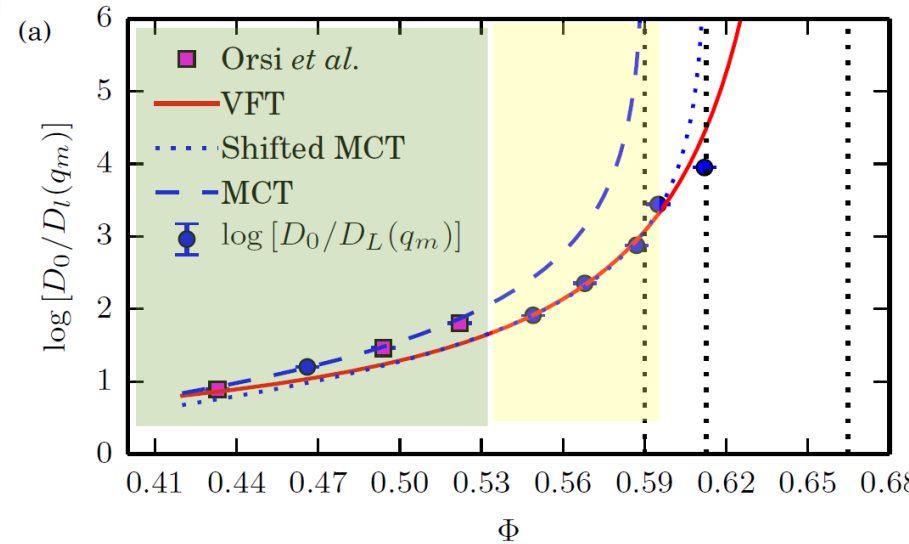
Age  $\sim 2$ h30

Age  $\sim 9$ h

Pawel Kwasniewski *et al.*

# Colloidal Glasses: Conclusions

- Low- $\Phi$ : Dynamics of colloids well explained by existing many-body theories (MCT)
- $\Phi \geq 0.57$ - $0.59$  Stress in the network and stress-induced (nonthermal) fluctuations become dominant and hinder the expected glass transition
- Non-equilibrium, complex dynamics determined by “rough” energy landscape (heterogeneities)  
*Hyperdiffusive relaxations*  
→ *jamming*  
(common also in other systems)
- Response to perturbations?  
→ *flow, shear*



# Acknowledgements

**Colloids** *Pawel Kwasniewski* (ESRF), *Davide Orsi* (U. Parma)  
A. Madsen (XFEL)

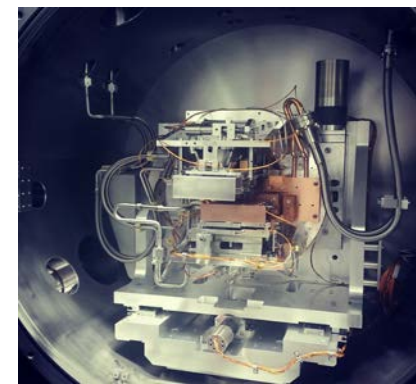
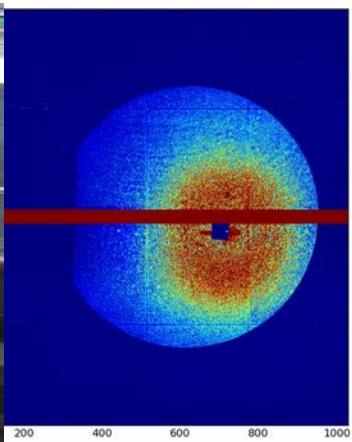
**Proteins** *Luxi Li*, V. Stojanoff, L. Wiegart (BNL), S. Mochrie (Yale)

**CHX** *Lutz Wiegart*, *Yugang Zhang*,  
M. Carlucci-Dayton, S. Antonelli, R. Greene,  
D. Chabot, W. Lewis,

**Beamlines** ID 10 ESRF - Y. Chushkin, 34-ID APS - R. Harder  
8-ID APS - A. Sandy, S. Narayanan

**NSLS-II** Ron Pindack, Qun Shen, P. Zschack, J. Hill, A. Broadbent  
O. Chubar, K. Evans-Lutterodt, P. Siddons ...

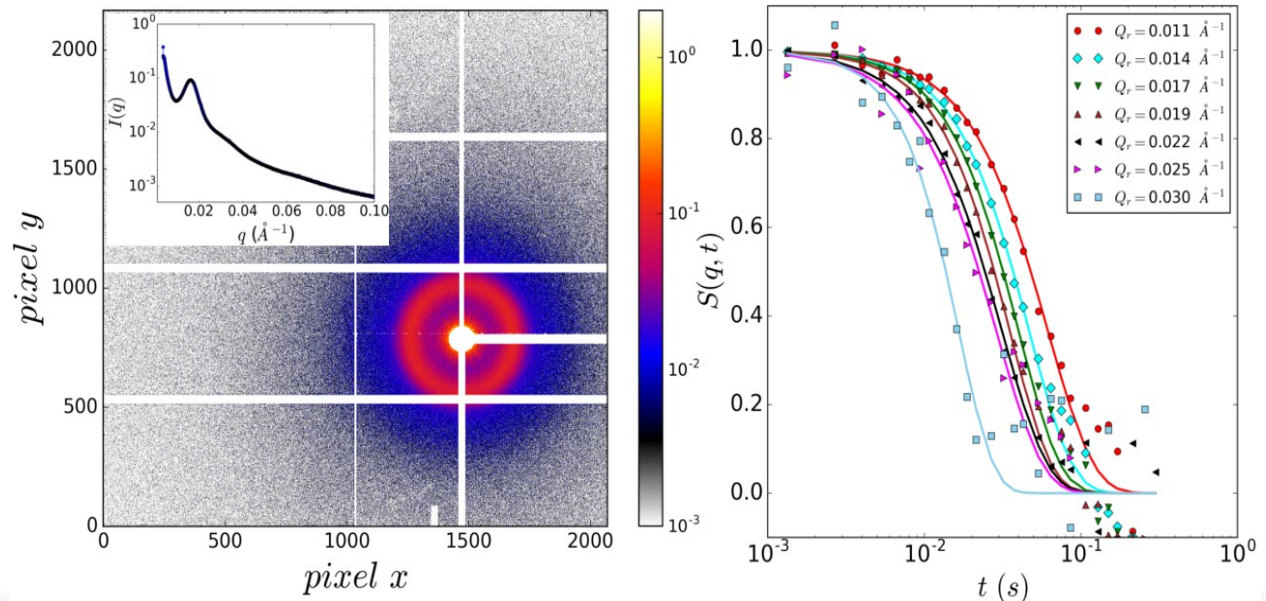
**Funding** NSLS-II project: DOE# E-AC02-98CH10886  
BNL SC0012704  
BNL LDRD 11-025



# A “Mini User Guide” to XPCS

## Questions:

- How much does the sample scatter?
  - we need  $\sim 10^{-N}$  ph/correlation time/speckle(pixel) -  $g^{(2)}$
  - We need  $\sim 1/\text{ph/correlation time/speckle(pixel)} - C(t_1, t_2)$
- What time scales are we expecting?
- What is the radiation limit? Is the sample homogeneous? i.e can we build an ensemble by averaging information recorded from different locations?

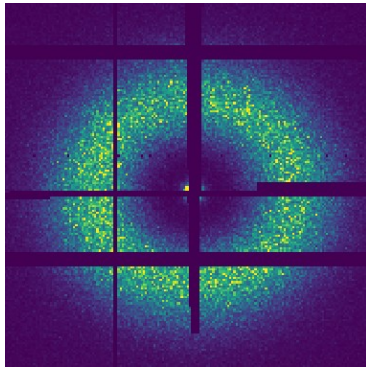


# Speckles

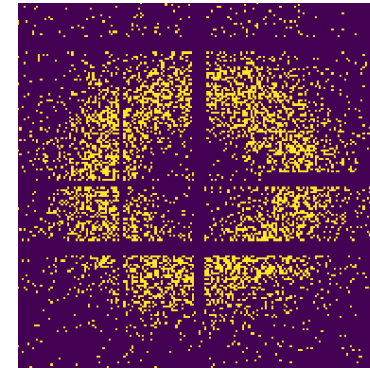
- Speckle statistics is described by the negative binomial distribution with
  - $M=M(q,T)$ : # of coherent modes
  - $K=K(q,T)$ : avg # of counts at a given q/ring
- Normalized variance becomes:

$$\text{var}_K(q,T) = \frac{1}{M(q,T)} + \frac{1}{K(q,T)}$$

Large  $K(q,T)$



Small  $K(q,T)$



Mandel, L. (1958). *Proc. Phys. Soc.* **72**, 1037.

Mandel, L. (1959). *Proc. Phys. Soc.* **74**, 233.

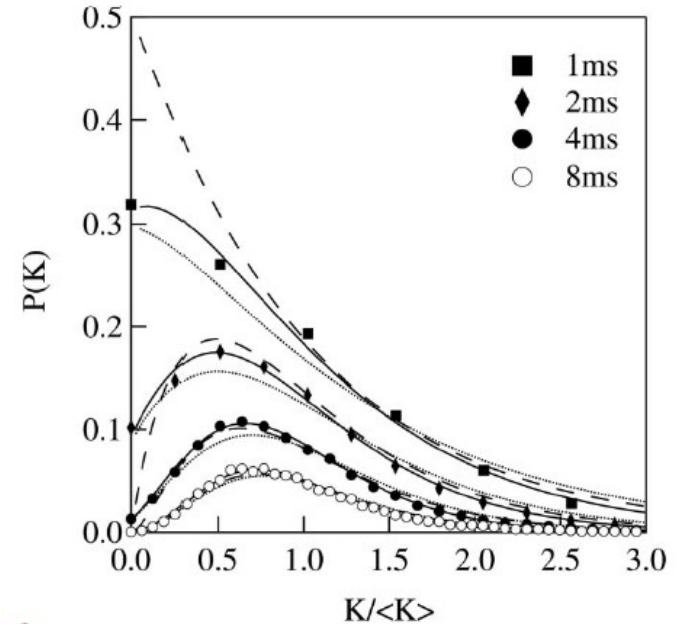
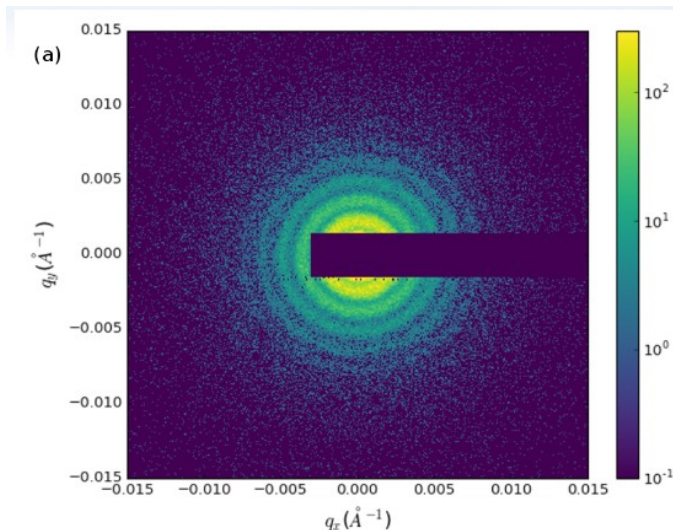
Goodman, J. W. (2007). *Speckle Phenomena in Optics: Theory and Applications*. Englewood: Roberts and Company.



# Speckles & Speckle Visibility Spectroscopy

- Speckle statistics is described by the negative binomial distribution with
  - $M=M(q, T)$ : # of coherent modes
  - $K=K(q, T)$ : avg # of counts at a given q/ring

$$P(K) = \frac{\Gamma(K + M)}{\Gamma(K + 1)\Gamma(M)} \left( \frac{M}{\langle K \rangle + M} \right)^M \left( \frac{\langle K \rangle}{M + \langle K \rangle} \right)^K$$



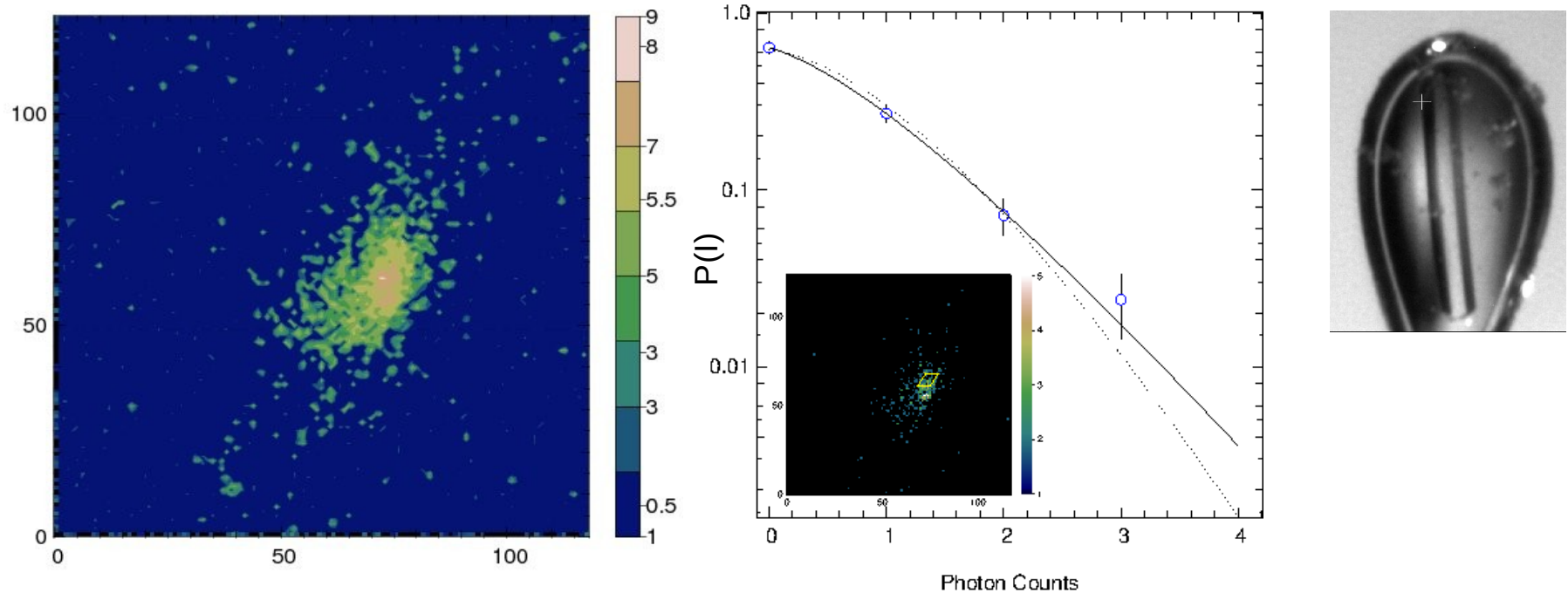
**Figure 2**

Photon count statistics analysis performed over an ensemble of pixels marked in the circular region in Fig. 1(a) for four integration times. Markers represent the photon count probability density  $P(K)$  from the experiments, and solid lines are the fitting curves using the negative-binomial distribution [equation (11)], dashed lines are the fitting curves using the gamma distribution [equation (5)] and dotted lines are the fits using equation (11) with  $M$  as the only fitting parameter, while  $\langle K \rangle$  is calculated from the measured photon counts. The results are plotted as a function of reduced count  $K/\langle K \rangle$ , so that  $P(K)$  values with different integration times can be stacked in the same figure.

*Luxi Li et al. J. Synch. Rad. 2014*

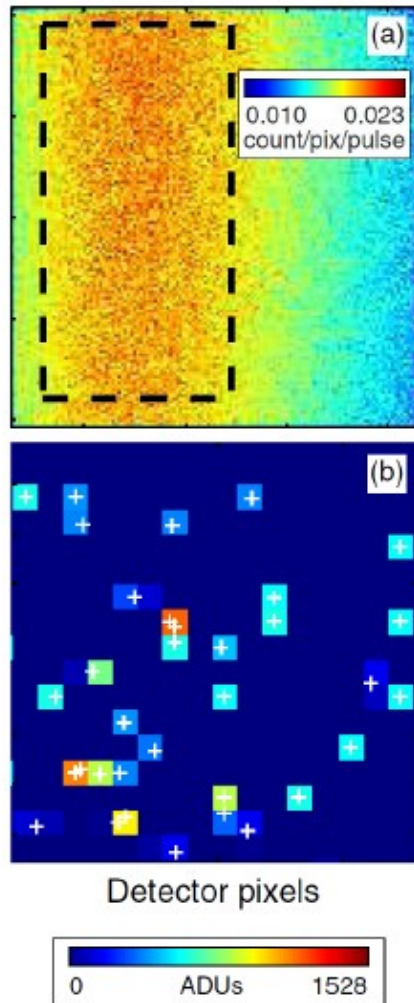
# X-ray Speckles come to life

- Molecular motion in protein microcrystals coupled over large scales generate diffuse scattering around the main Bragg peaks.

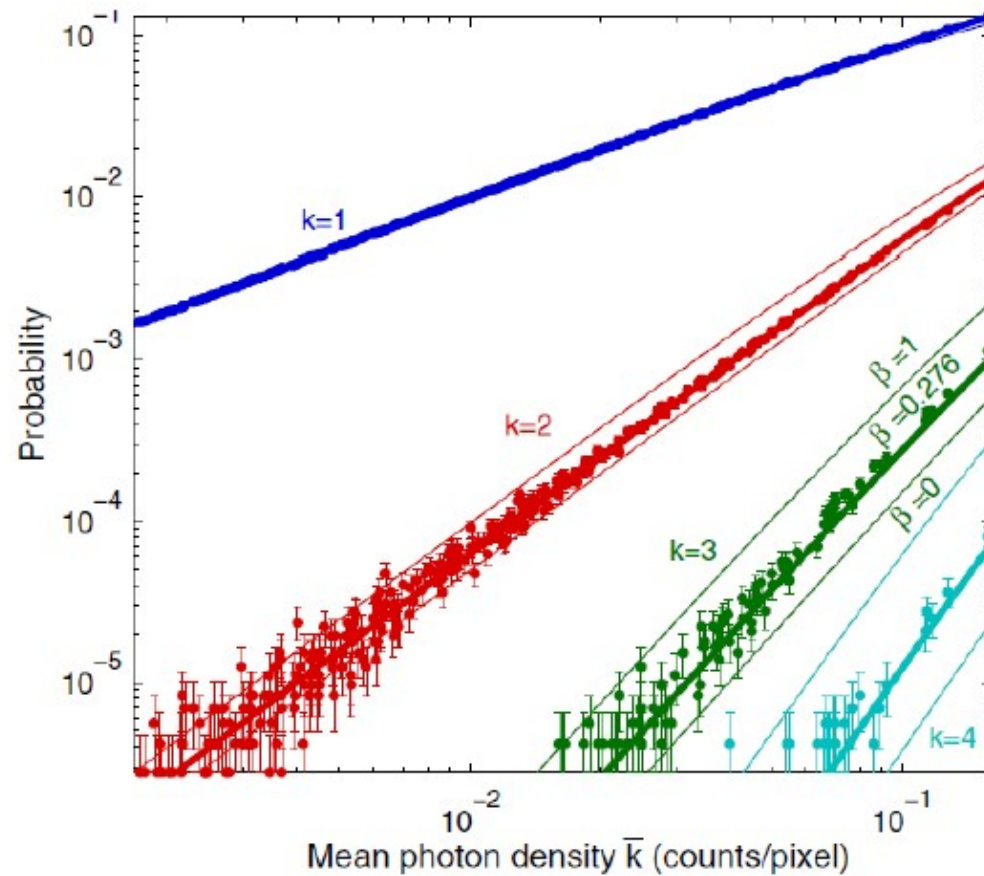


L. Li *et al.*, unpublished

# Speckles from single shot LCLS pulses



Single-shots at LCLS, Poisson-Gamma statistics



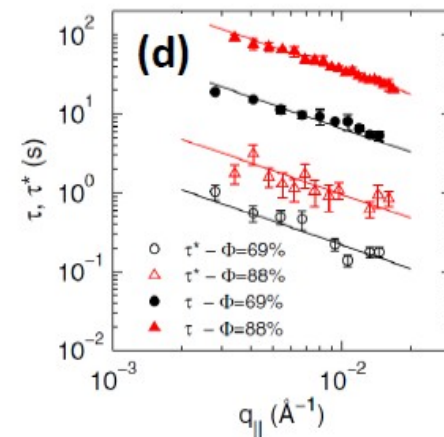
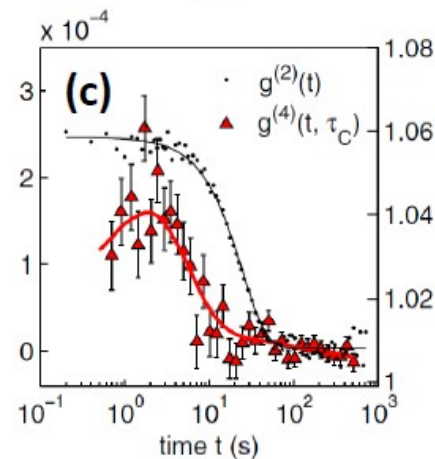
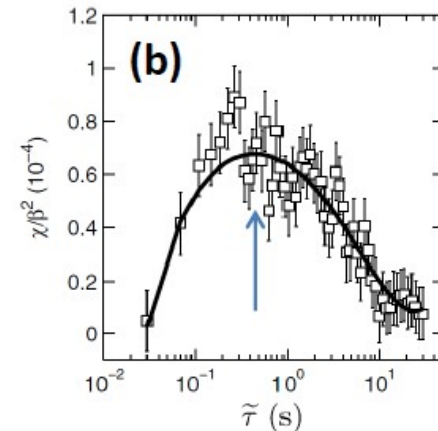
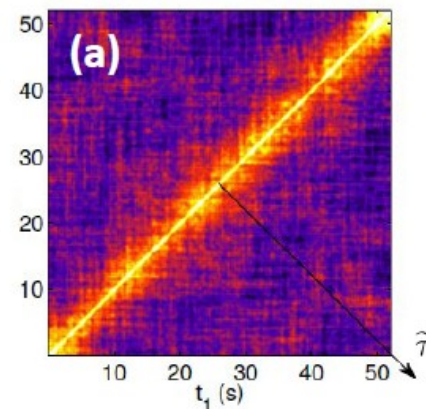
S. O. Hruszkewycz et al., PRL109, 185502 (2012)

# 4<sup>th</sup> order correlations: dynamical heterogeneities

- Orsi et al. - dynamics in langmuir monolayer of nanoparticles using Grazing Incidence (GI)-XPCS
- Heterogeneities (correlations of correlations)

$$g^{(4)}(t, \tilde{\tau}) = \langle C(t_1, t_1 + \tilde{\tau}) C(t_1 + t, t_1 + t + \tilde{\tau}) \rangle_{t_1}$$

$$= \langle I(t_1) I(t_1 + \tilde{\tau}) I(t_1 + t) I(t_1 + t + \tilde{\tau}) \rangle_{t_1}$$



A. Duri *et al.*, *Phys. Rev. E* **72**, 051401 (2005)

D. Orsi *et al.*, *Phys. Rev. Lett.* **108**, 105701 (2012)