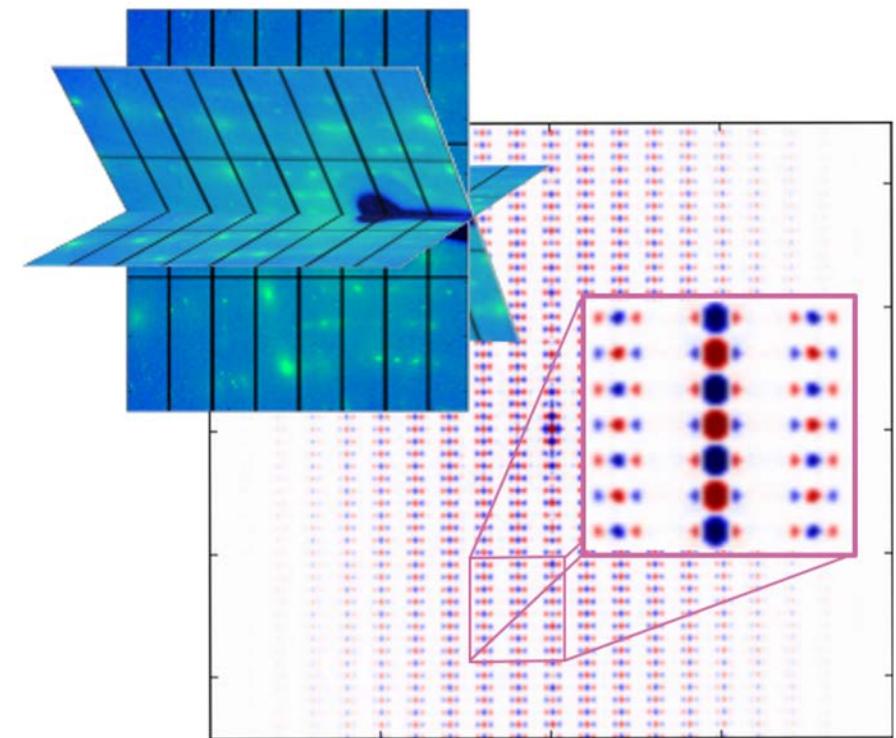


Single Crystal Diffuse Scattering

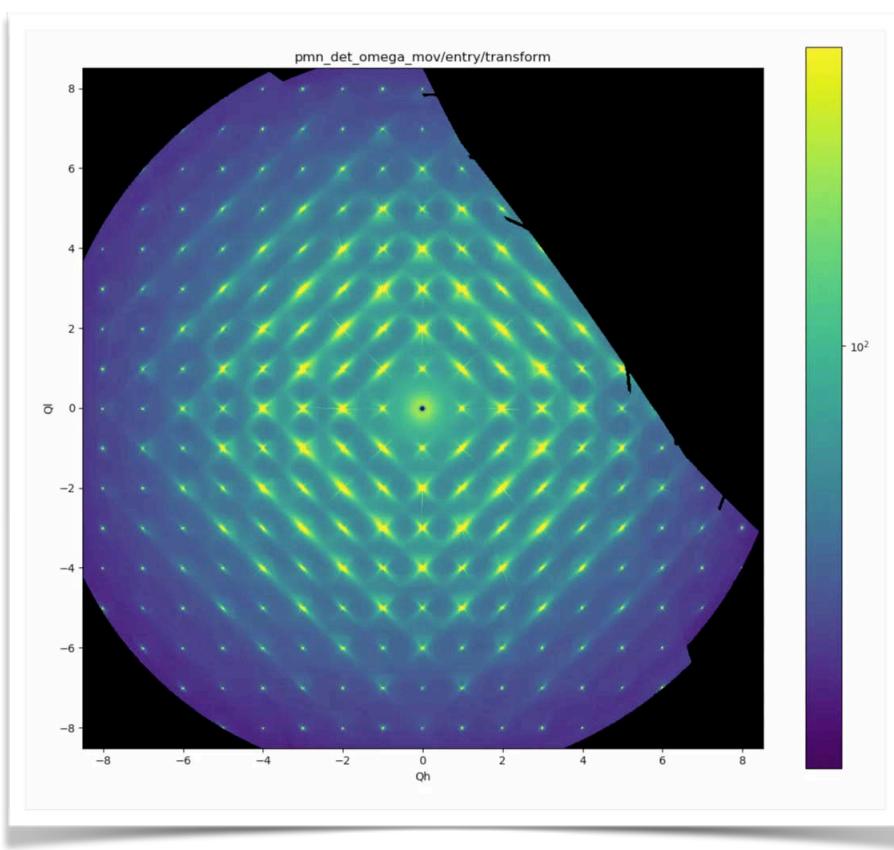
Ray Osborn

Neutron and X-ray Scattering Group
Materials Science Division
Argonne National Laboratory



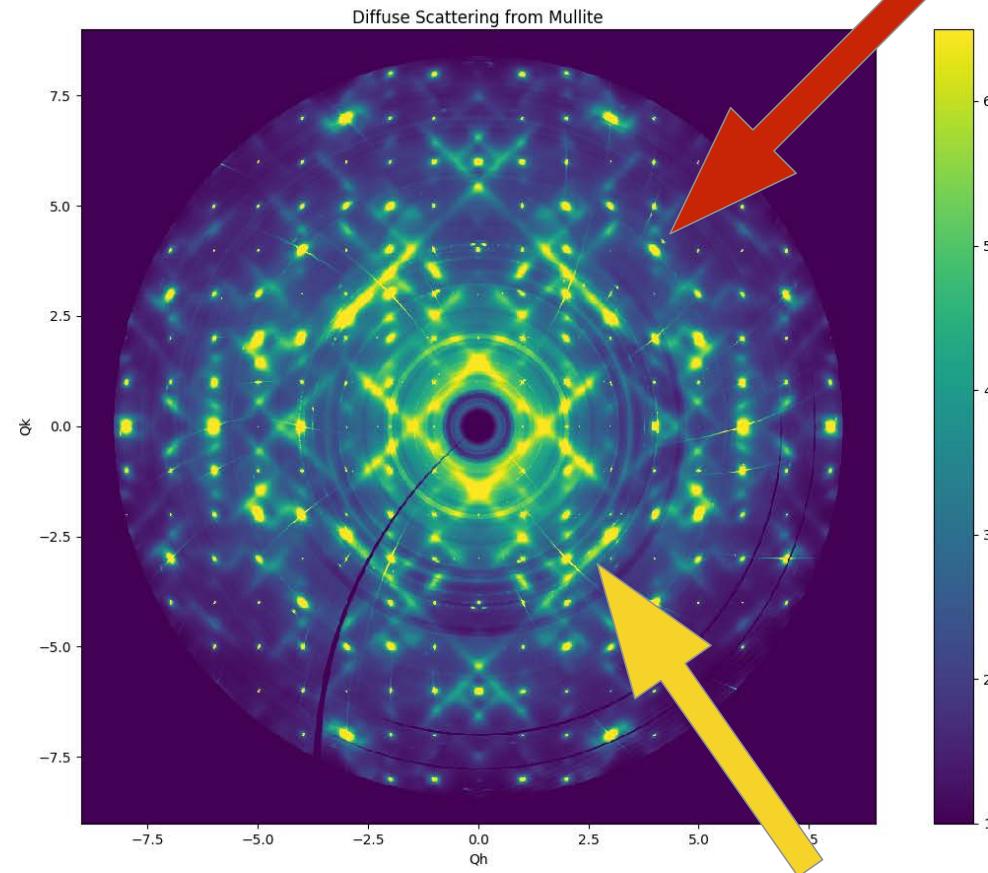
Outline

- ▶ What is diffuse scattering?
 - What does it look like?
 - What causes it?
 - Who started it?
- ▶ What is it good for?
 - A random walk through disordered materials
- ▶ How do I model it?
 - A few equations
 - Rules of thumb
- ▶ Case Study 1: Diffuse scattering from vacancies in mullite
- ▶ Case Study 2: 3D- Δ PDF in sodium-intercalated V_2O_5
- ▶ How do I look at static disorder? *Hint: Corelli*
 - Neutrons vs X-rays
 - Diffuse scattering with elastic discrimination
- ▶ Diffuse scattering - the musical



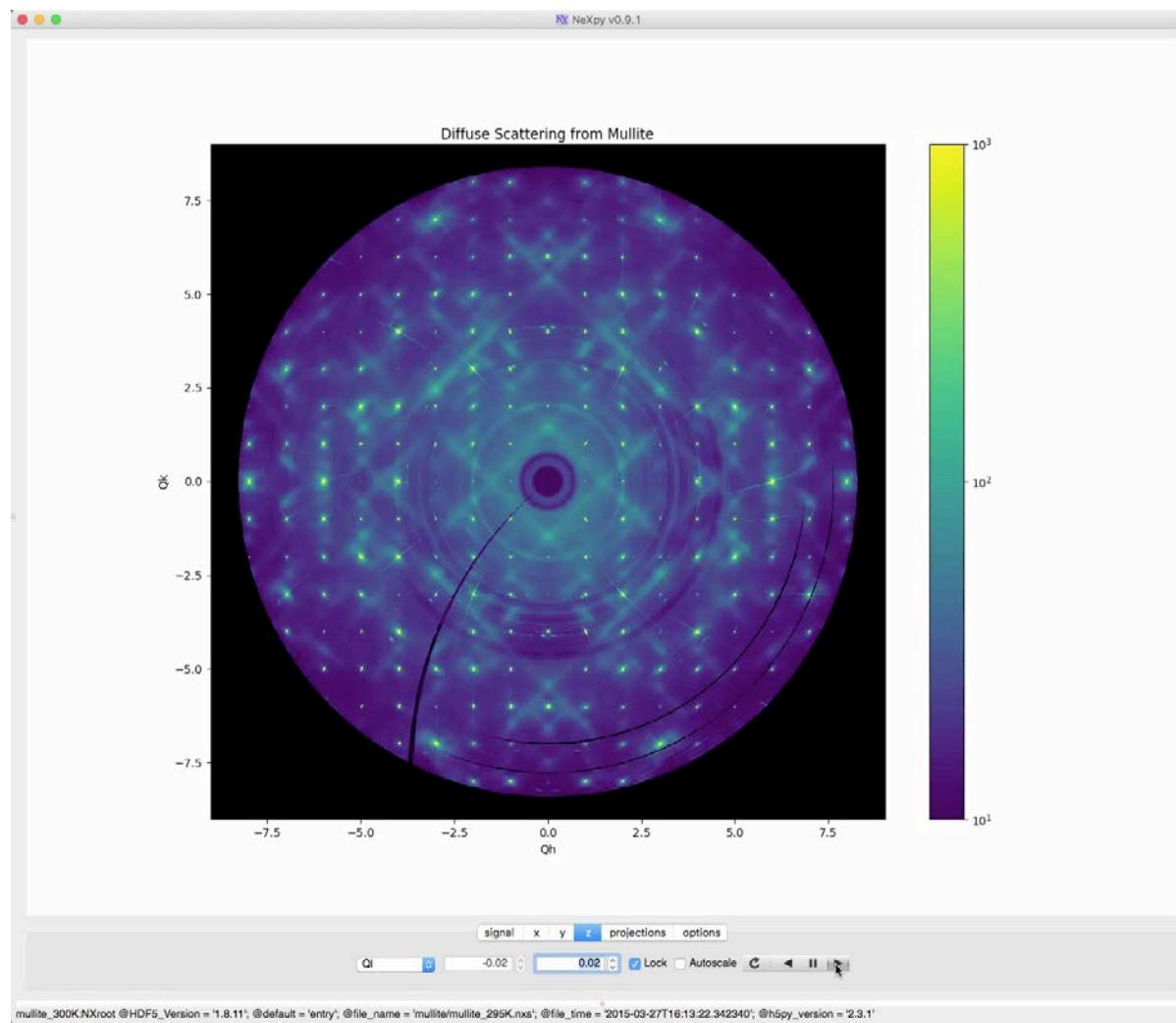
Diffuse Scattering

Bragg Scattering Average Structure



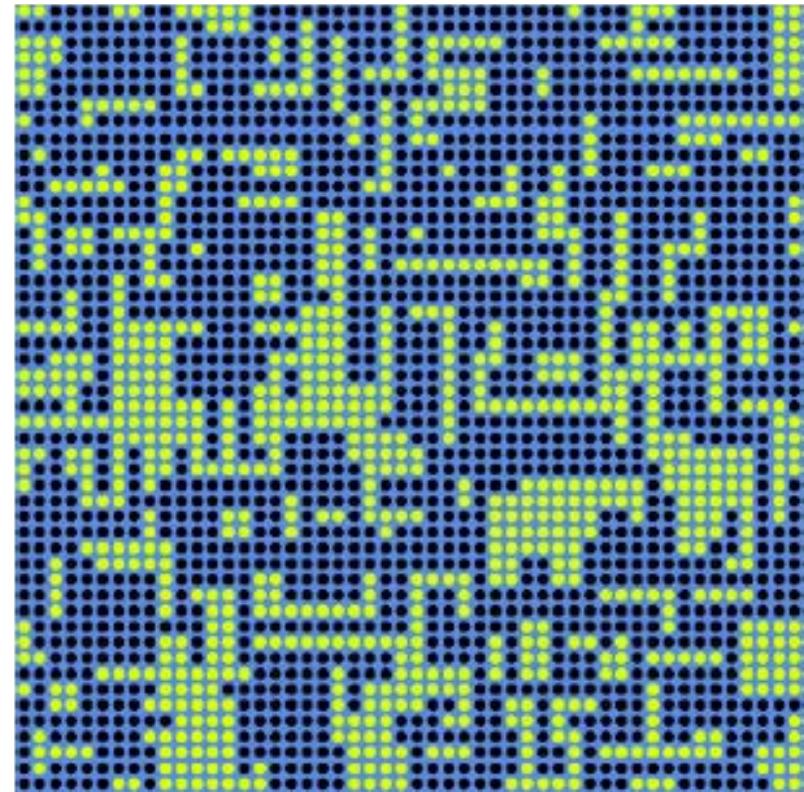
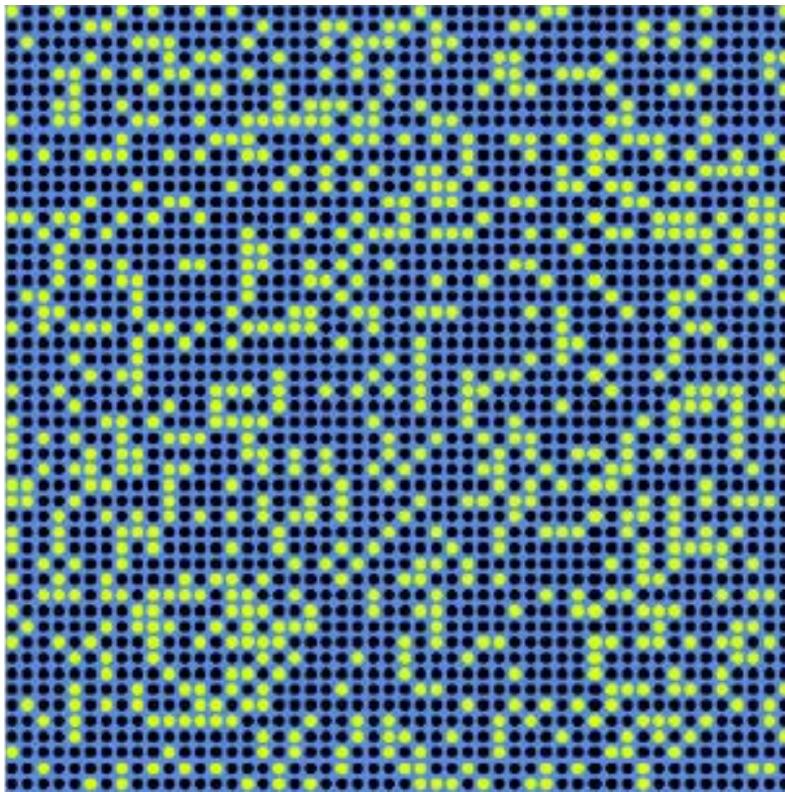
Diffuse Scattering
Deviations from the Average Structure

Single Crystal Diffuse Scattering in 3D



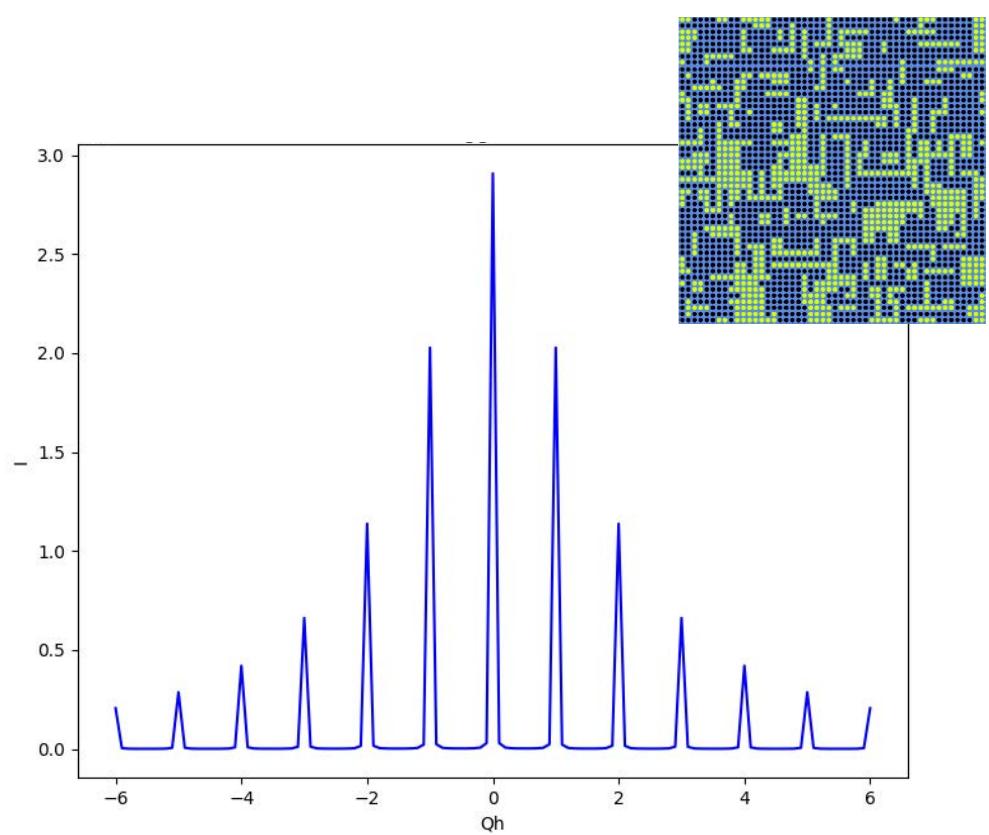
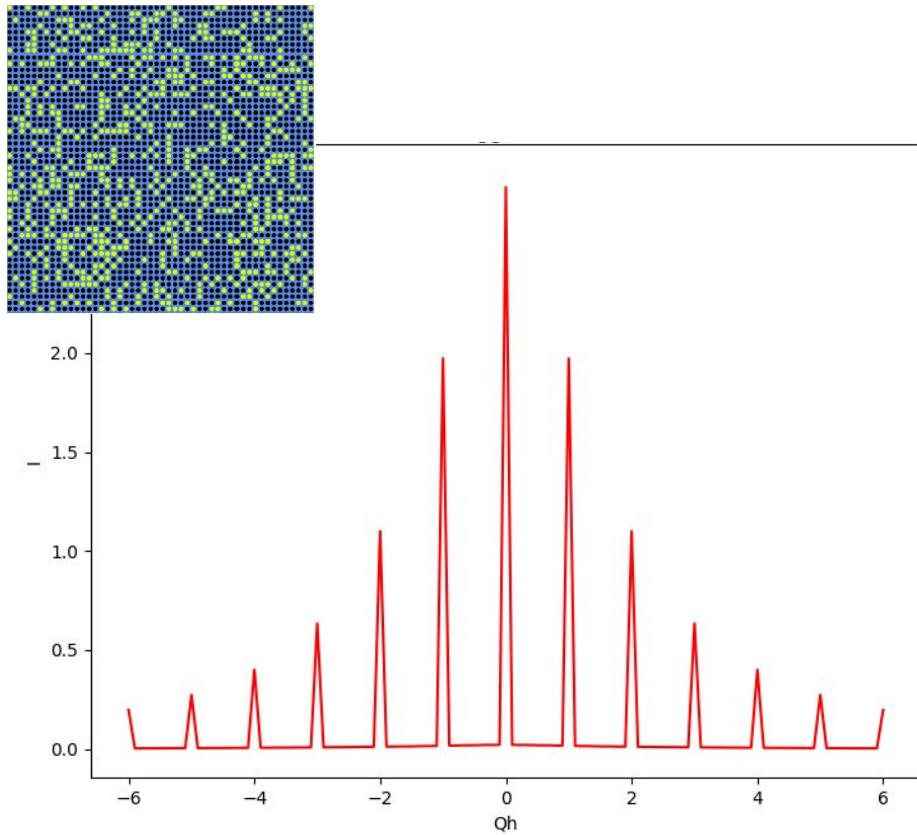
Simple Example of Disorder

- ▶ In these examples, 30% of atoms (blue dots) have been replaced by vacancies (green dots)
 - Left-Hand-Side: random substitution
 - Right-Hand-Side: high probability of vacancy clusters
- Thanks to Thomas Proffen



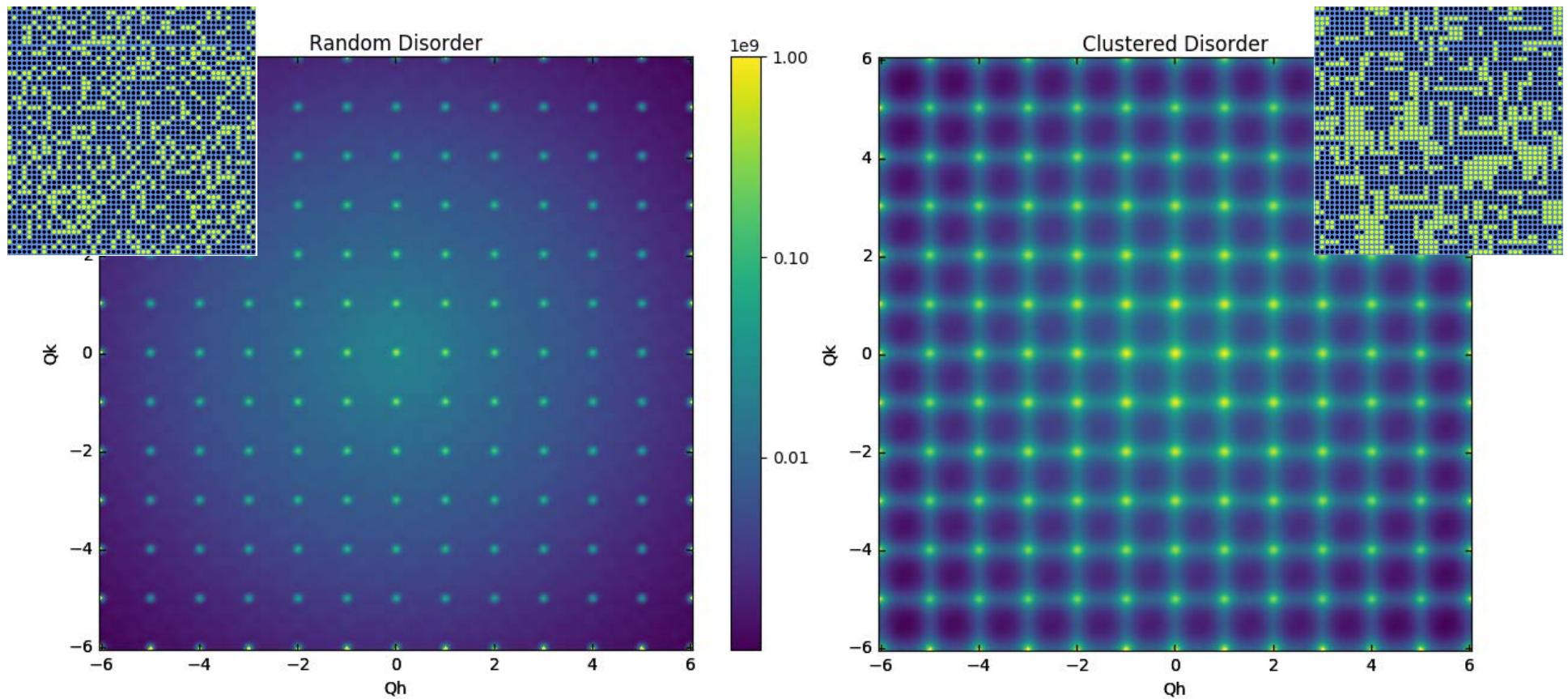
Bragg Scattering

- ▶ Bragg scattering is determined by the average structure.
 - Since the average vacancy occupation is identical, both examples have identical Bragg peaks

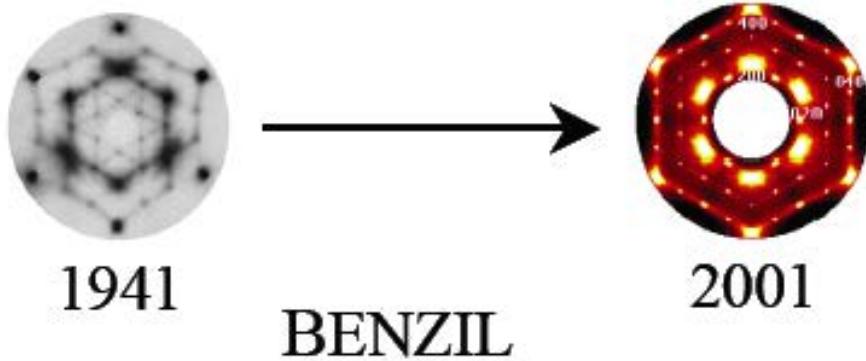


Diffuse Scattering

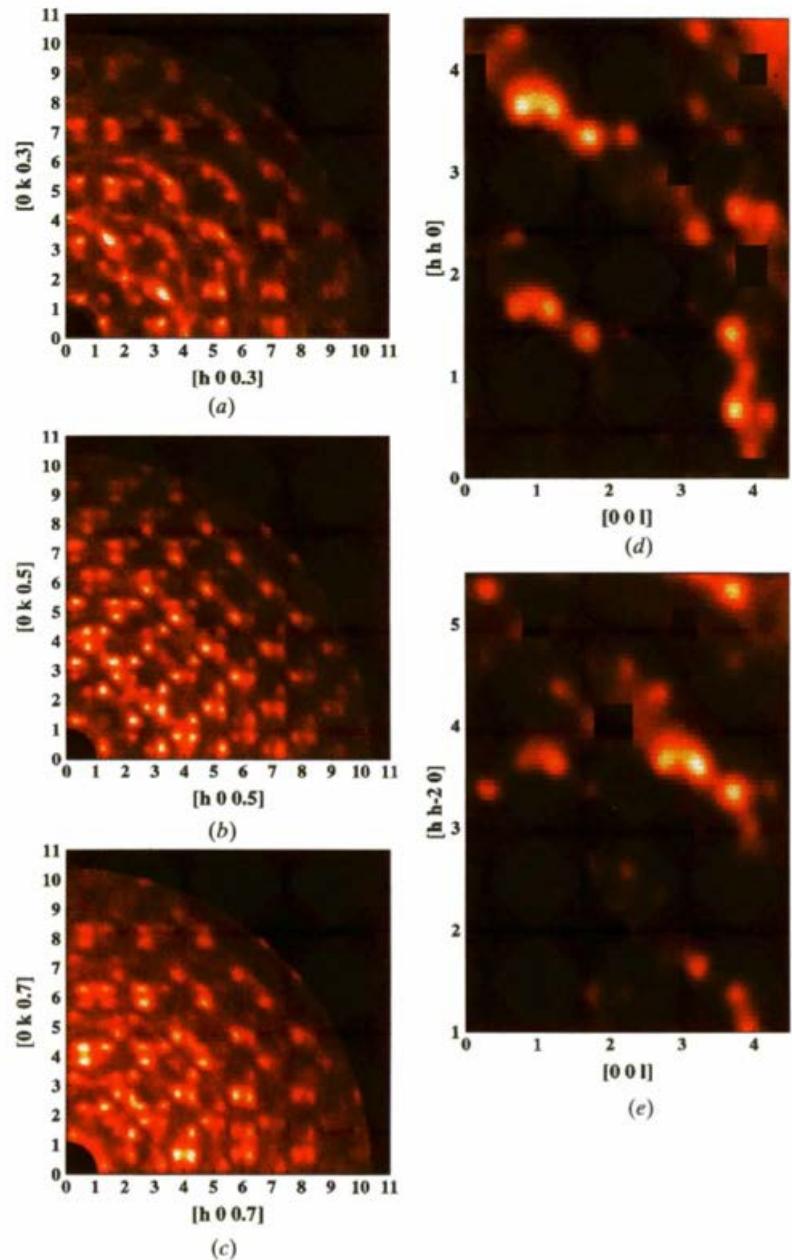
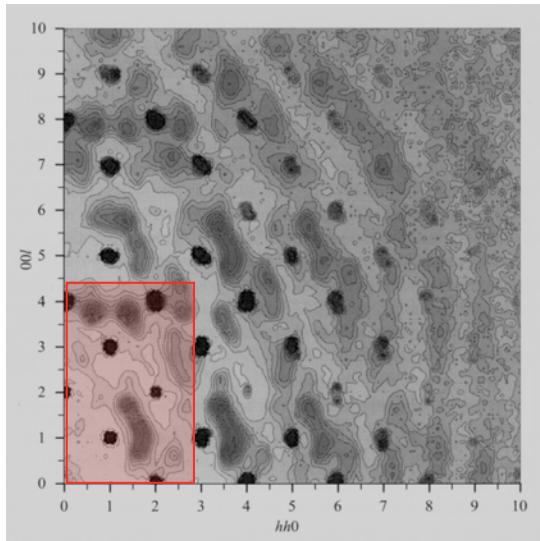
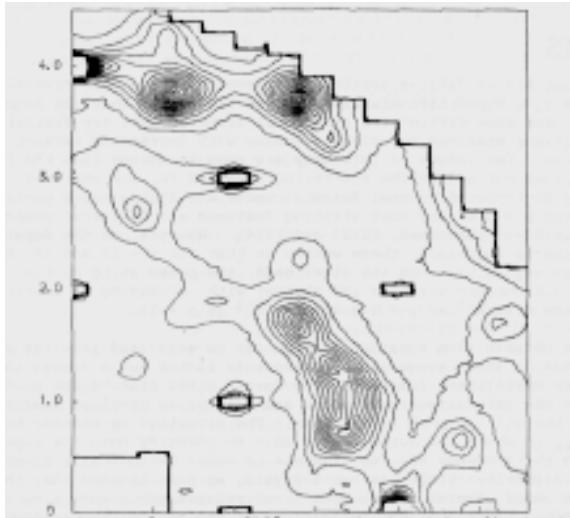
- ▶ The diffuse scattering is quite different in the two examples
 - Random vacancy distributions lead to a constant background (Laue monotonic scattering)
 - Vacancy clusters produce rods of diffuse scattering connecting the Bragg peaks



An Ultra-Short History of Advances in Diffuse Scattering



Yttria-Stabilized Zirconia



T. Proffen and T. R. Welberry J. Appl. Cryst. 31, 318 (1998)

What is it good for?

Science Impacted by Diffuse Scattering

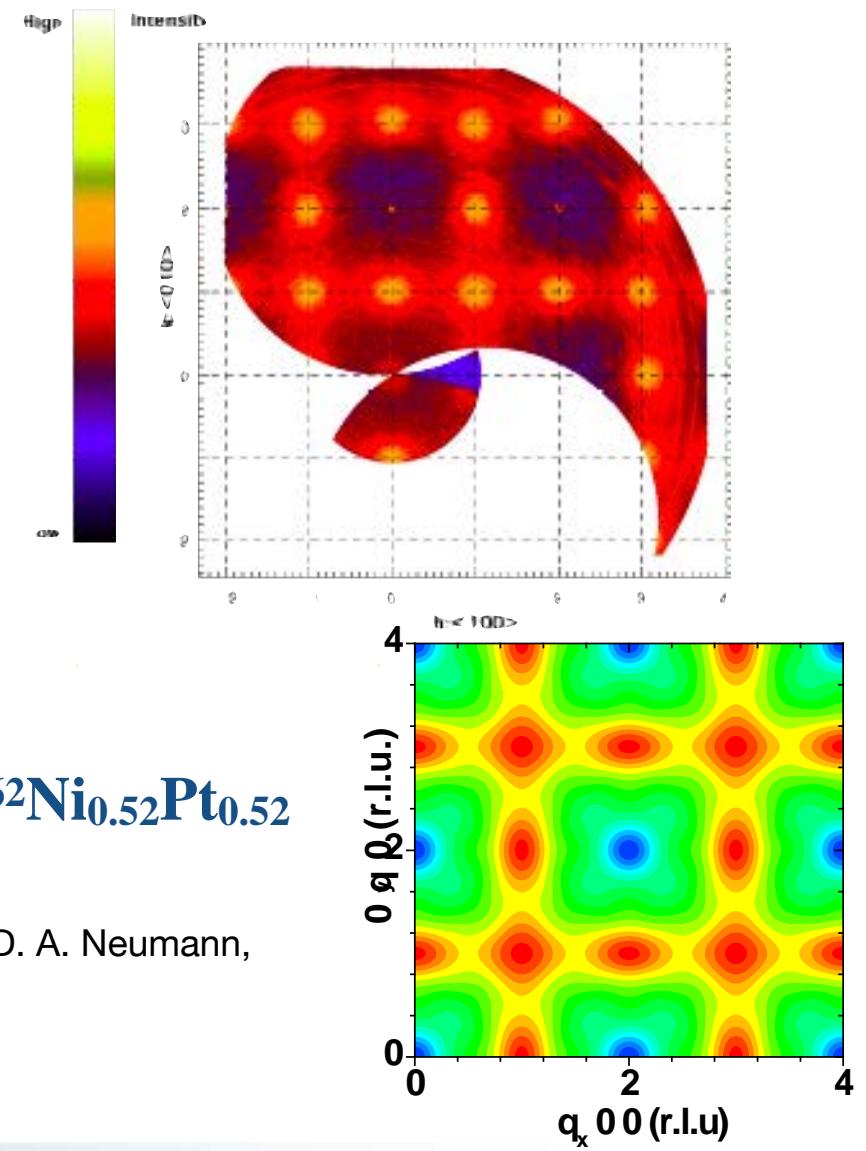
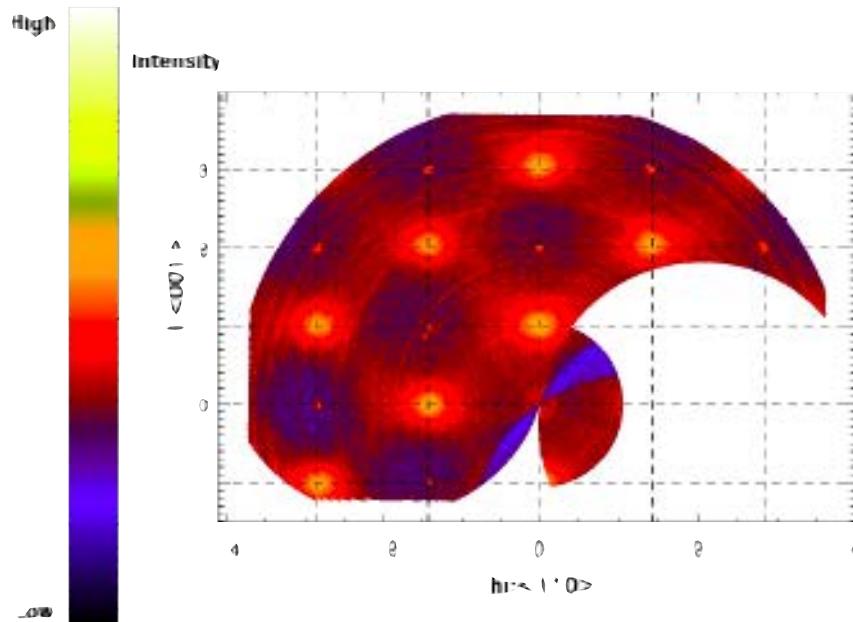
► Subjects identified at the *Workshop on Single Crystal Diffuse Scattering at Pulsed Neutron Sources*

- Stripes in cuprate superconductors
- Orbital correlations in transition metal oxides (including CMR)
- Nanodomains in relaxor ferroelectrics
- Defect correlations in fast-ion conductors
- Geometrically frustrated systems
- Critical fluctuations at quantum phase transitions
- Orientational disorder in molecular crystals
- Rigid unit modes in framework structures
- Quasicrystals
- Atomic and magnetic defects in metallic alloys
- Molecular magnets
- Defect correlations in doped semiconductors
- Microporous and mesoporous compounds
- Host-guest systems
- Hydrogen-bearing materials
- Soft matter - protein configurational disorder using polarization analysis of spin-incoherence
- Low-dimensional systems
- Intercalates
- Structural phase transitions in geological materials



<<http://www.neutron.anl.gov/diffuse/>>

Diffuse Scattering from Metallic Alloys

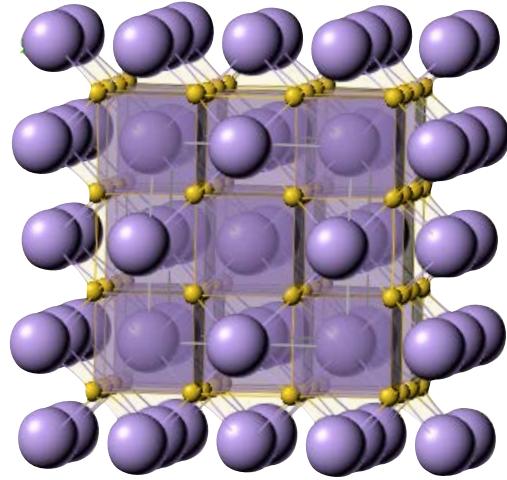


Short-range Order in Null Matrix $^{62}\text{Ni}_{0.52}\text{Pt}_{0.52}$

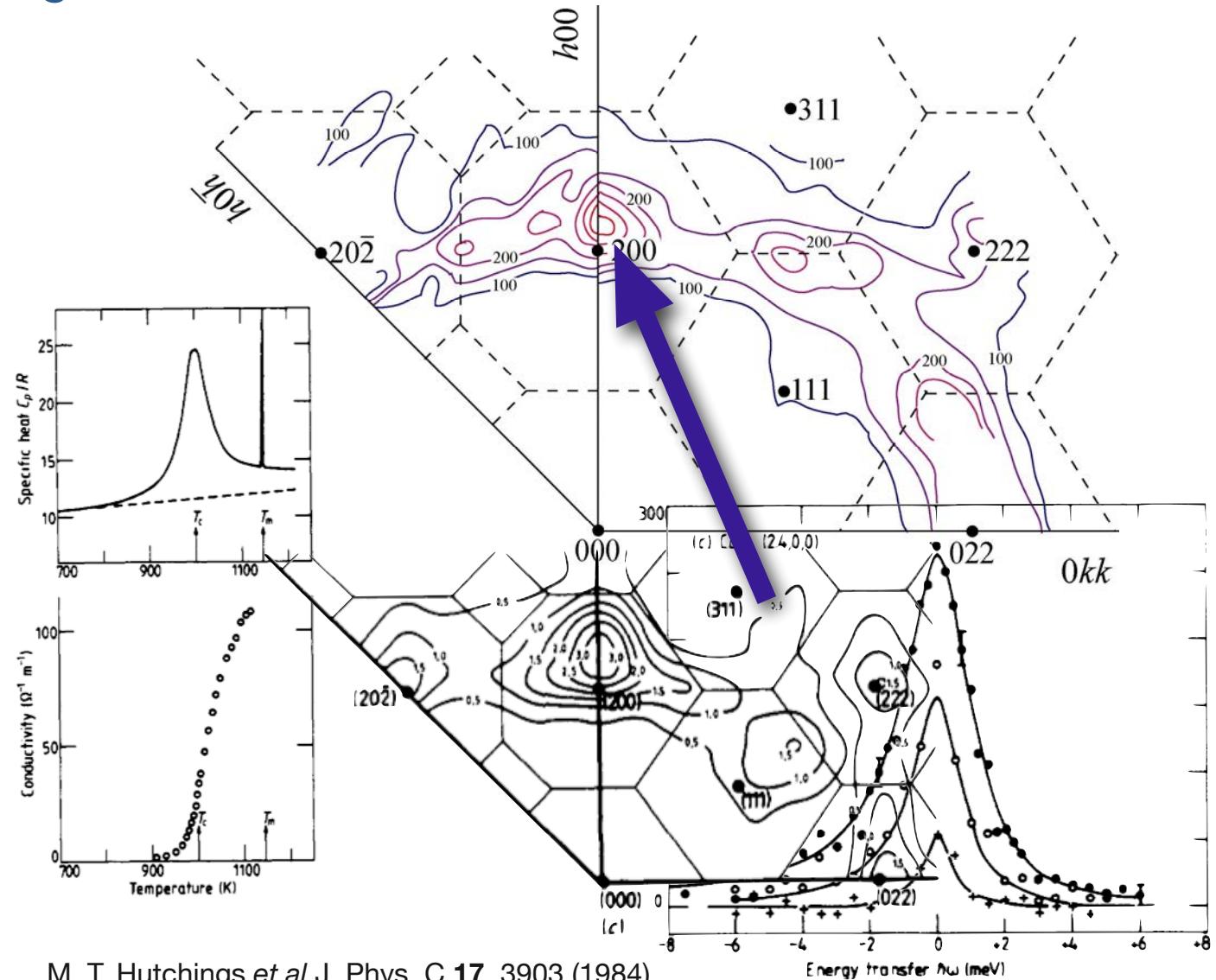
J. A. Rodriguez, S. C. Moss, J. L. Robertson, J. R. D. Copley, D. A. Neumann, and J. Major

Phys. Rev. B **74**, 104115

Diffuse Scattering from a Fast-Ion Conductor

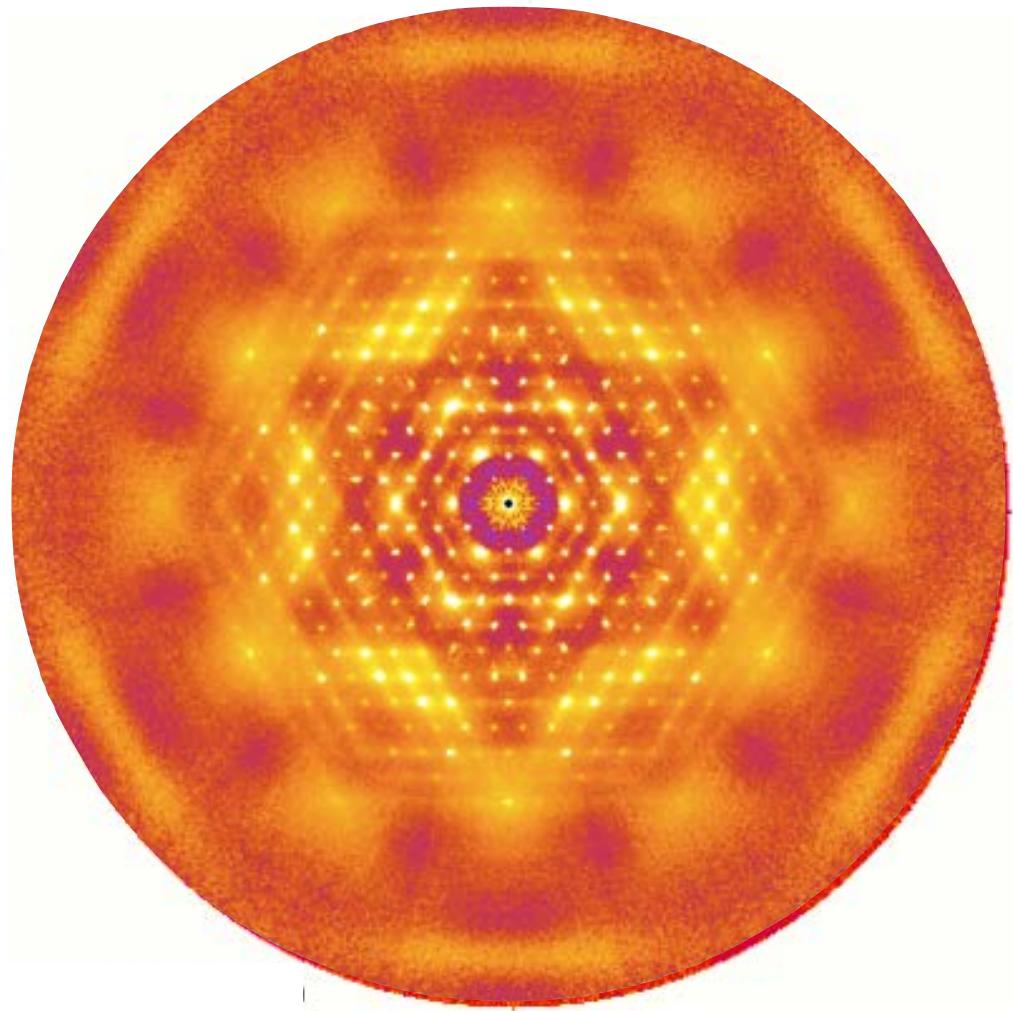
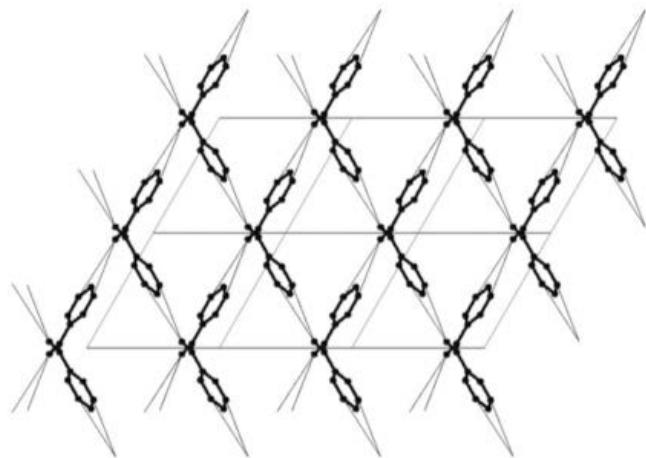
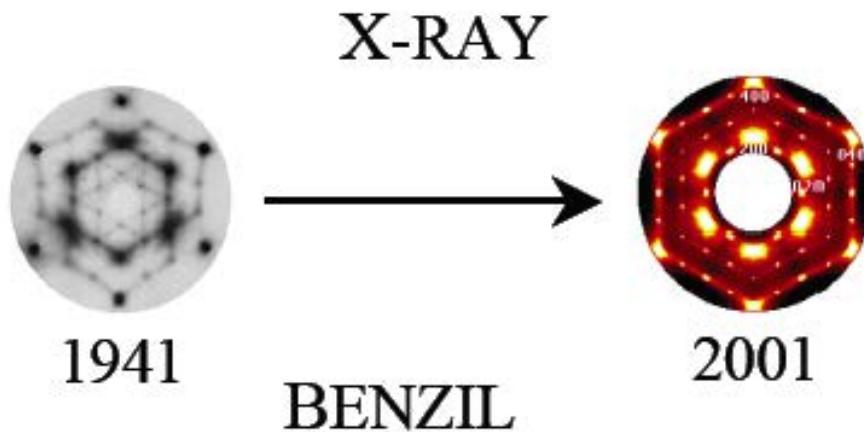


CaF₂



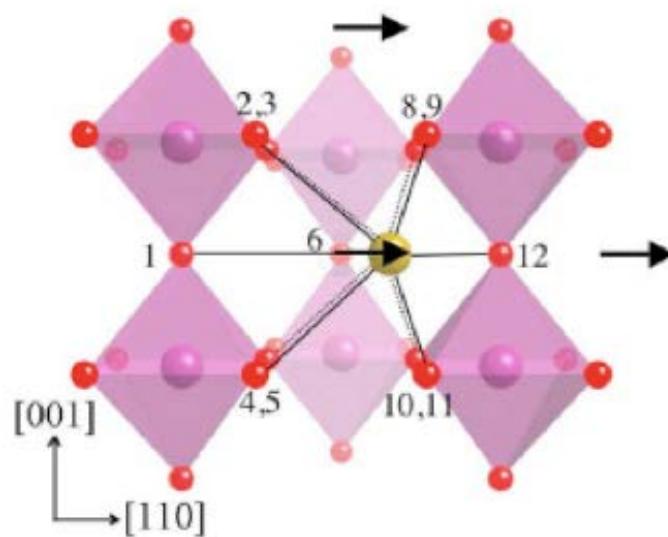
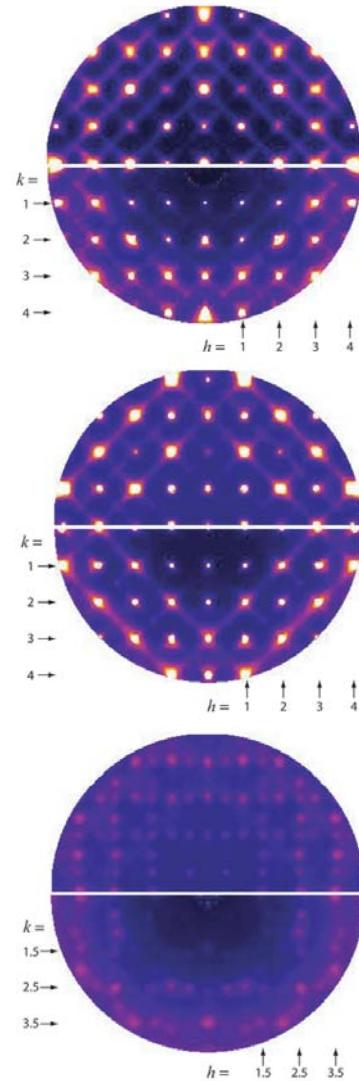
M. T. Hutchings et al J. Phys. C 17, 3903 (1984)

Diffuse Scattering from Molecular Solids

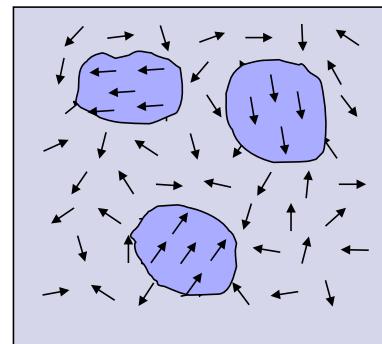


T. R. Welberry *et al* J. Appl. Cryst. **36**, 1400 (2003)

Diffuse Scattering from Relaxor Ferroelectrics



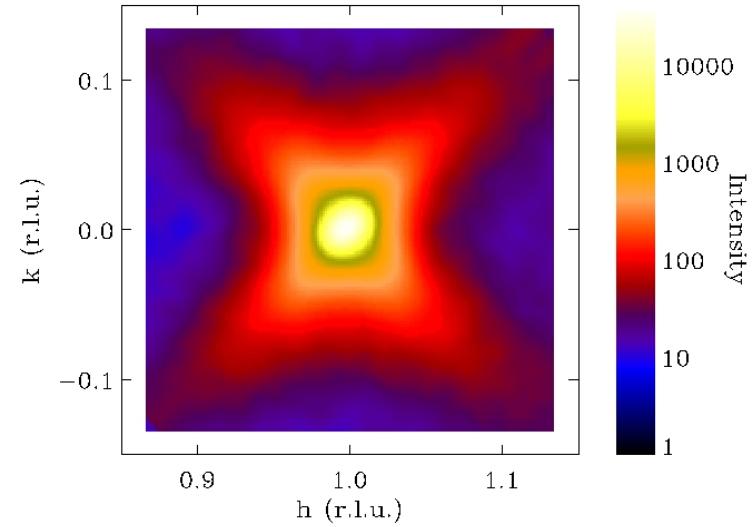
Lead Zinc-Niobate



T. R. Welberry et al J. Appl. Cryst. **38**, 639 (2005)

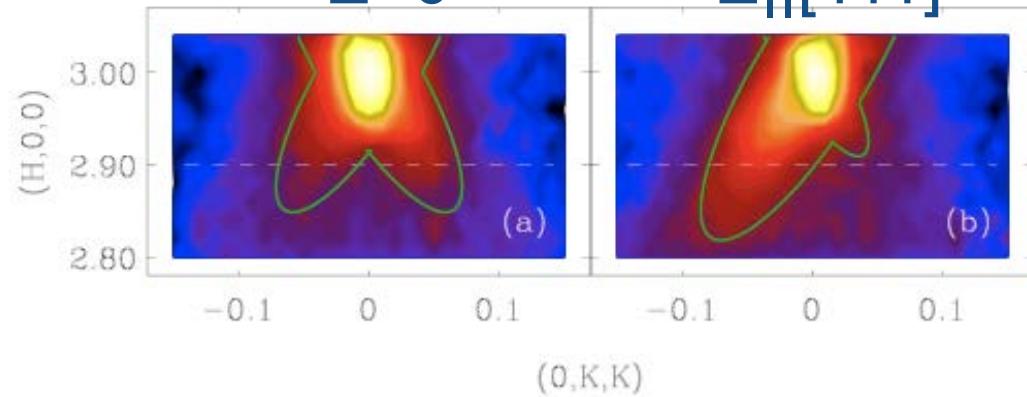
National School on Neutron & X-ray Scattering - 2019

Lead Magnesium-Niobate



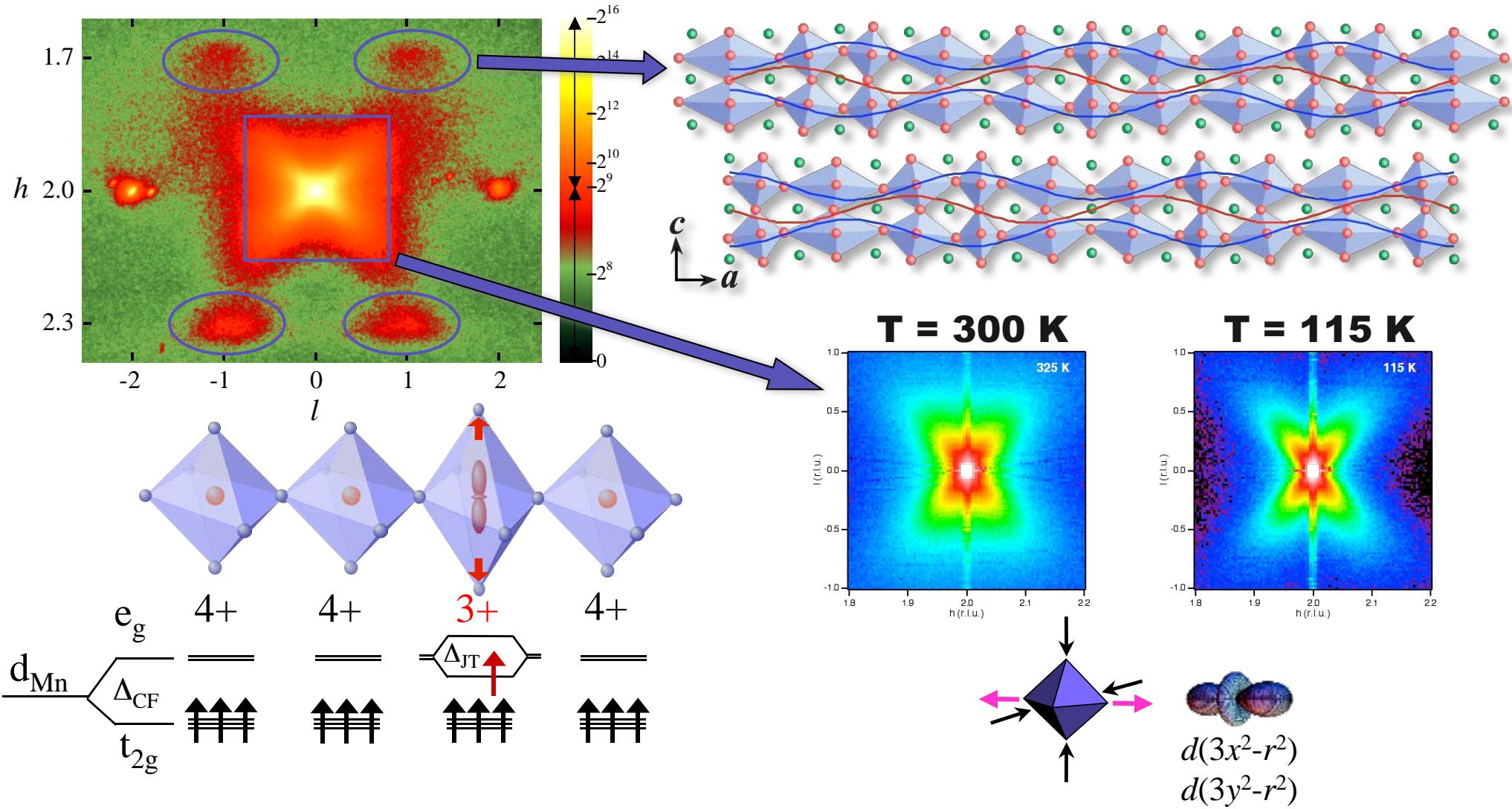
$E=0$

$E \parallel [111]$

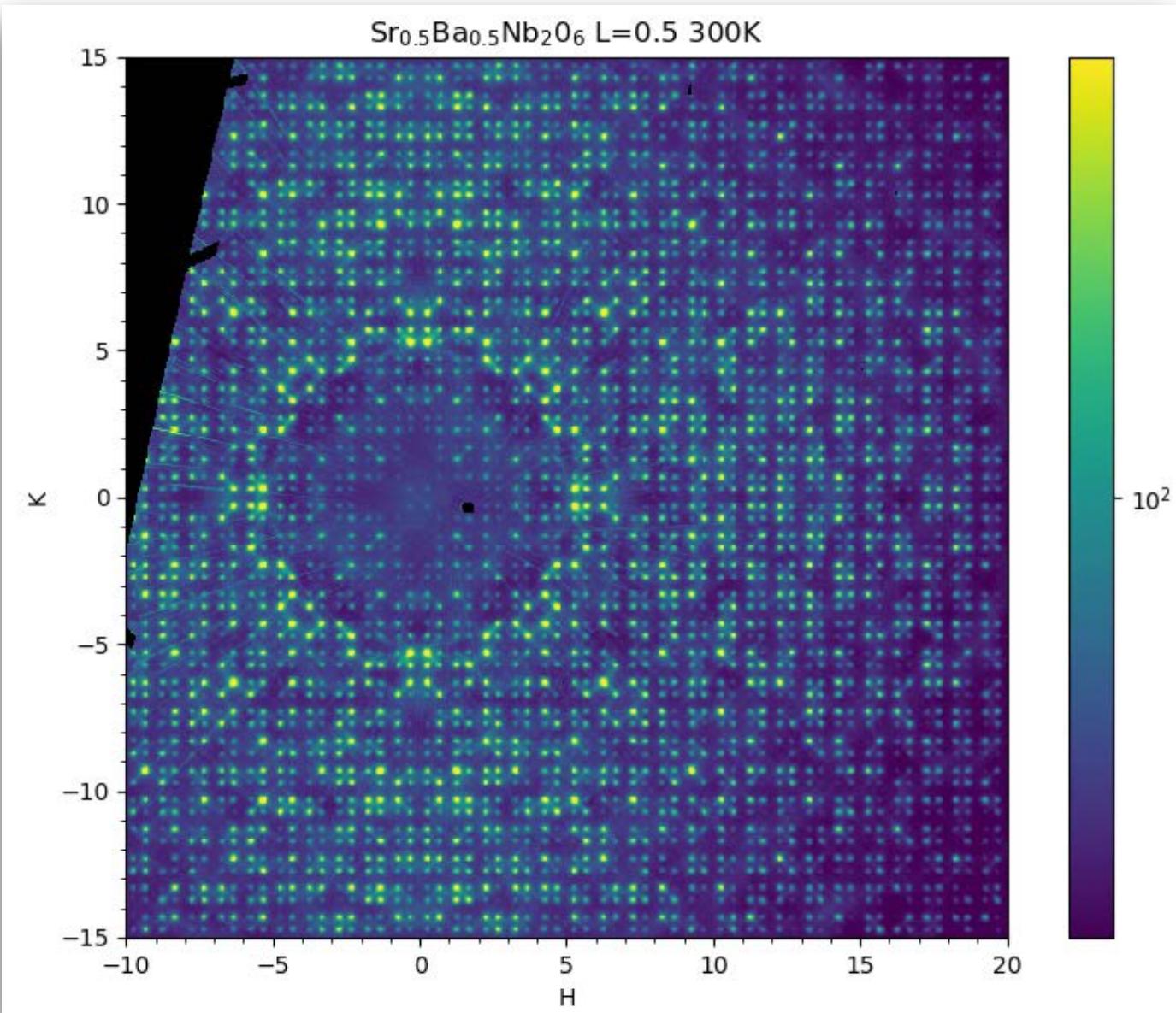
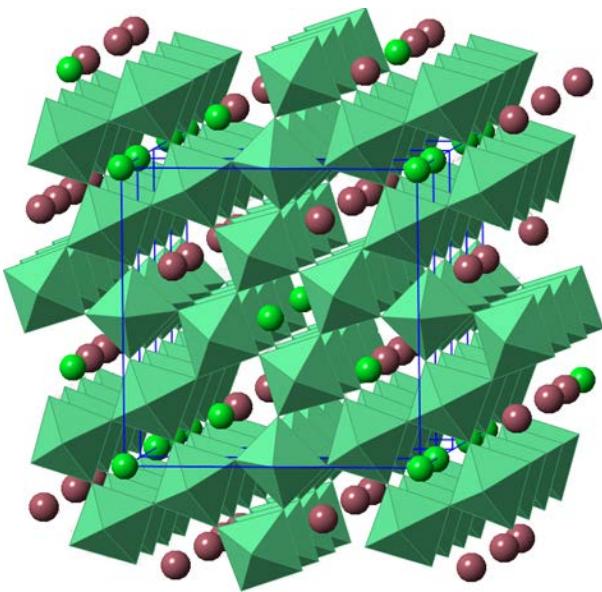


G. Xu, P. M. Gehring, G. Shirane, Phys. Rev. B **72**, 214106 (2005).

Diffuse Scattering from Jahn-Teller Polarons

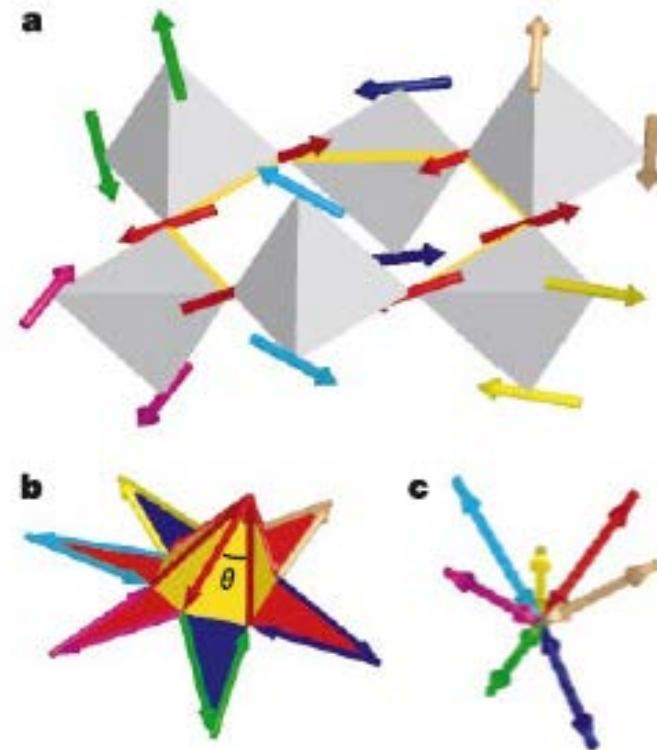
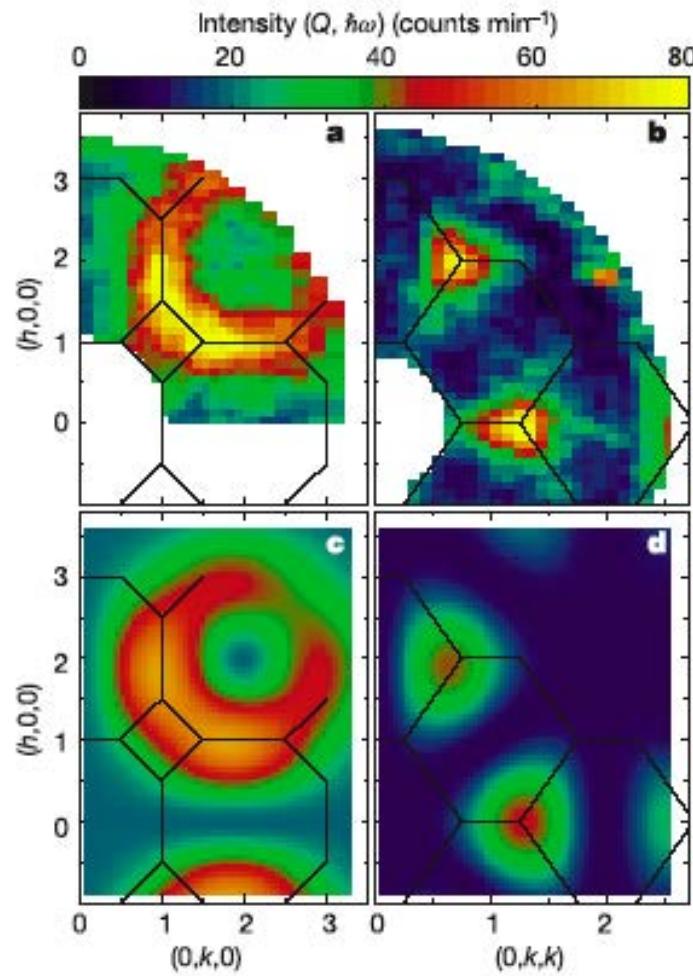


Incommensurate Modulations in $\text{Sr}_{0.5}\text{Ba}_{0.5}\text{NbO}_6$



Acknowledgements:
Bixia Wang and Daniel Phelan

Magnetic Diffuse Scattering from Geometric Frustration



ZnCr₂O₄

S.-H. Lee *et al* Nature **418**, 856 (2002)

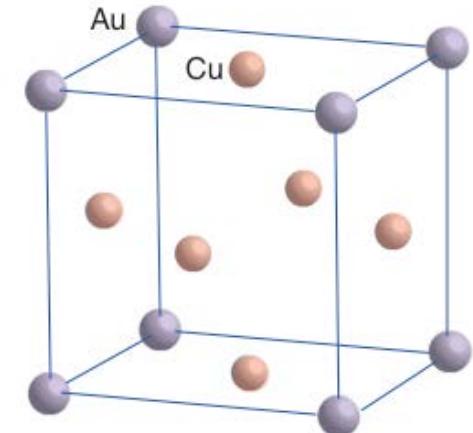
How do I model it?

A Few Equations

V. M. Nield and D. A. Keen *Diffuse Neutron Scattering From Crystalline Materials* (2001)
T. R. Welberry *Diffuse X-ray Scattering and Models of Disorder* (2004)

$$I = \sum_i \sum_j b_i b_j \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij})$$

Laue Monotonic Diffuse Scattering



J. M. Cowley, J. Appl. Phys. **21**, 24 (1950)

$$I = \bar{b}^2 \sum_{ij} \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}) + N(\overline{b^2} - \bar{b}^2); \quad \bar{b}^2 = (c_A b_A + c_B b_B)^2; \quad \overline{b^2} = c_A c_B (b_B - b_A)^2$$

Cowley Short-Range Order

$$I_{diffuse} = N c_A c_B (b_B - b_A)^2 + \sum_{ij} \alpha_{ij} c_B c_A (b_B - b_A)^2 \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}); \quad \alpha_{ij} = \left(1 - \frac{P_{ij}}{c_j}\right)$$

Warren Size Effect

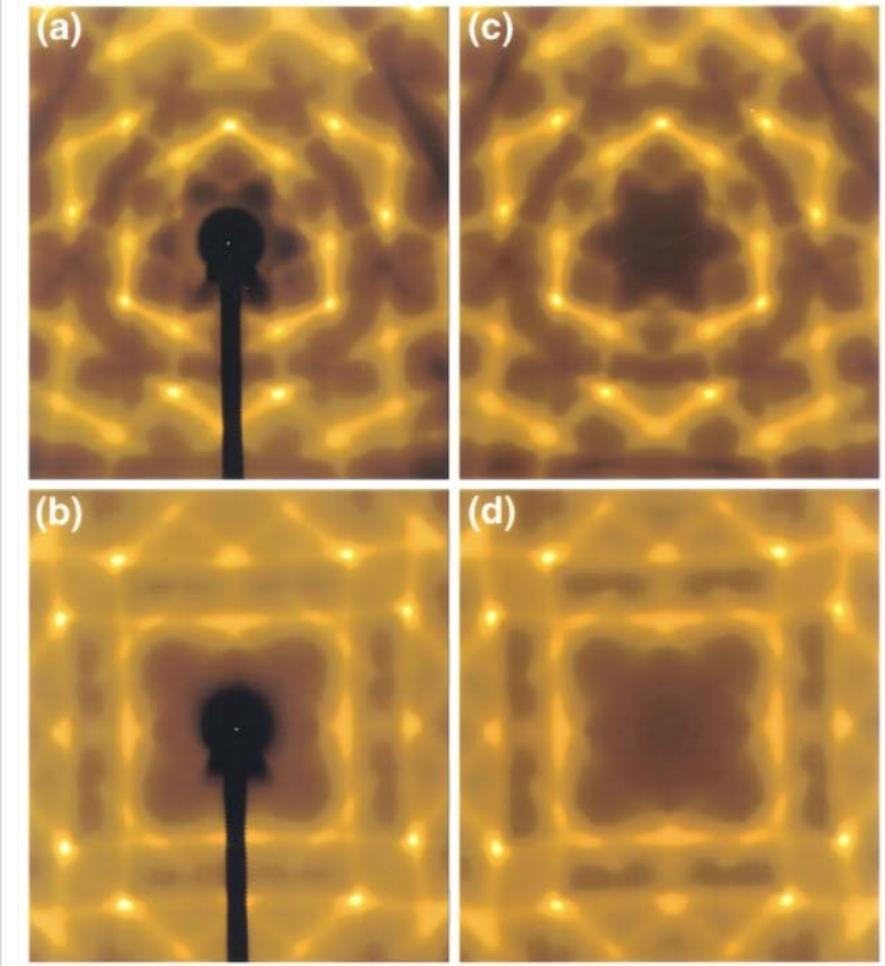
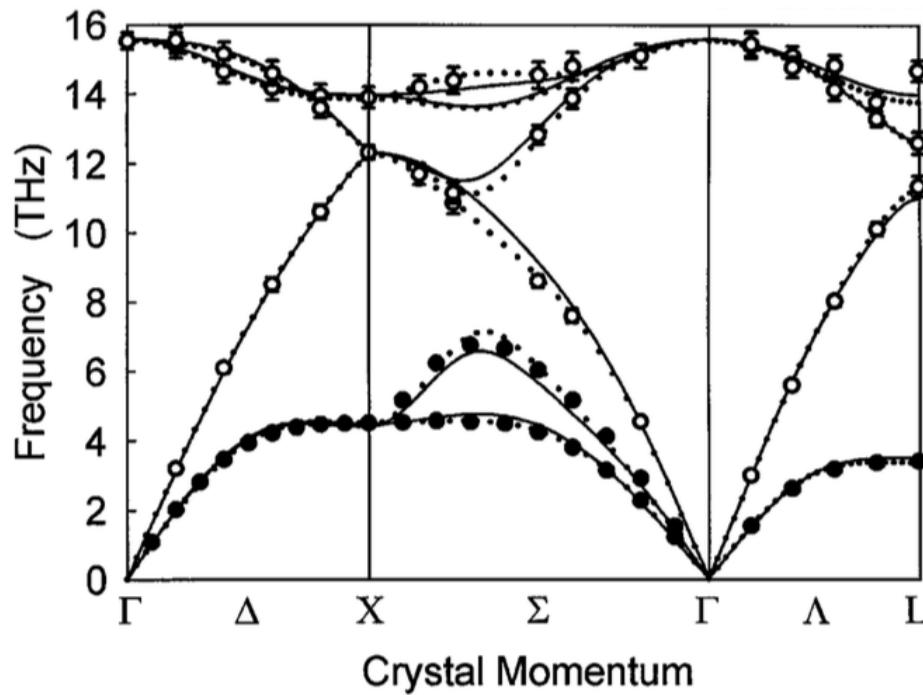
$$I_{diffuse} = N c_A c_B (b_B - b_A)^2 \left(1 + \sum_{ij} \alpha_{ij} \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}) + \beta_{ij} \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}) \right); \quad \beta_{ij} = f(\epsilon_{AA}^{ij}, \epsilon_{BB}^{ij})$$

Borie and Sparks Correlations

$$I = \sum_i \sum_j b_i b_j \exp(i\mathbf{Q} \cdot (\mathbf{R}_i - \mathbf{R}_j)) \left[1 + i\mathbf{Q} \cdot (\mathbf{u}_i - \mathbf{u}_j) - \frac{1}{2} (\mathbf{Q} \cdot (\mathbf{u}_i - \mathbf{u}_j))^2 + \dots \right]$$

Thermal Diffuse Scattering

- Lattice vibrations produce deviations from the average structure even in perfect crystals
- X-ray scattering intensity is given by the integral over all the phonon branches at each \mathbf{Q}



$$I_0 \propto f^2 e^{-2M} \sum_{j=1}^6 \frac{|\mathbf{q} \cdot \hat{\mathbf{e}}_j|^2}{\omega_j} \coth\left(\frac{\hbar\omega_j}{2k_B T}\right).$$

M. Holt, et al, Phys Rev Lett **83**, 3317 (1999).

Some Rules of Thumb (thanks to Hans Beat Bürgi)

Reciprocal space

- Only sharp Bragg reflections



Direct space

- 3D-periodic structure

- no defects

- Sharp diffuse rods



- 2D-periodic structure

- perpendicular to the streaks
 - disordered in streak directions

- Sharp diffuse planes



- 1D-periodic structure

- perpendicular to the planes
 - disordered within the plane

- Diffuse clouds



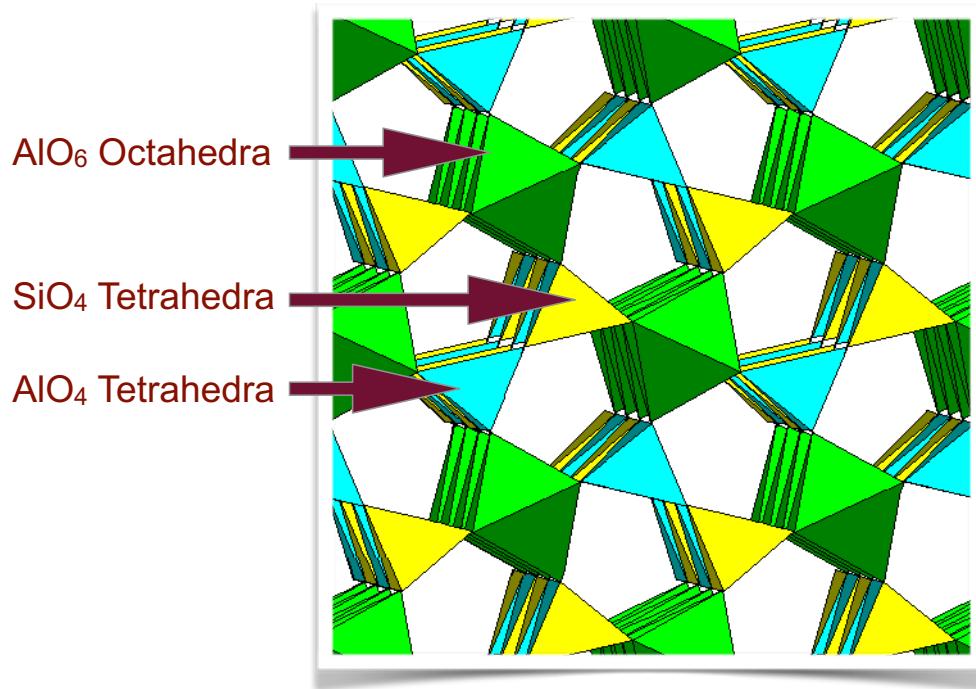
- 0D-periodic structure

- no fully ordered direction

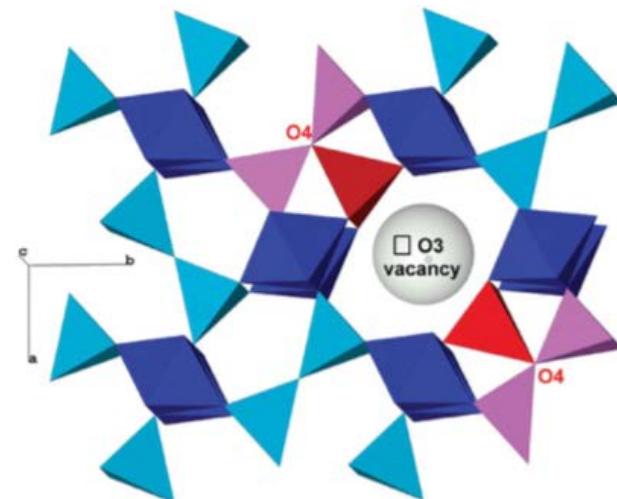
Case Study 1: Mullite

Mullite - A Case Study

- Mullite is a ceramic that is formed by adding O²⁺ vacancies to Sillimanite
 - Sillimanite has alternating AlO₄ and SiO₄ tetrahedra
 - Mullite has excess Al³⁺ occupying Si²⁺ sites for charge balance
- This results in strong vacancy-vacancy correlations



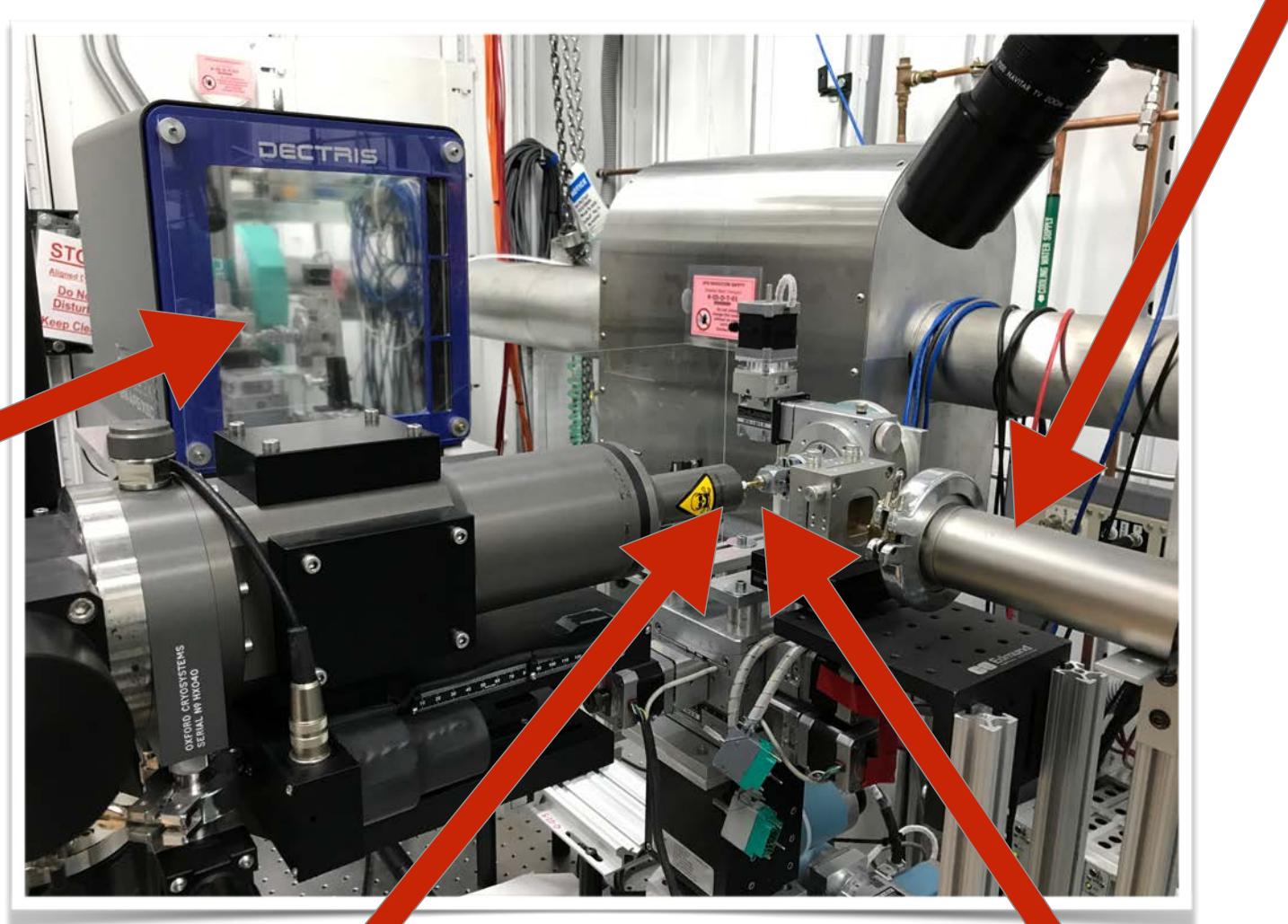
Sillimanite: Al_2SiO_5



Mullite: $\text{Al}_2(\text{Al}_{2+2x}\text{Si}_{2-2x})\text{O}_{10-x}$

Measuring X-ray Diffuse Scattering with Continuous Rotation Method

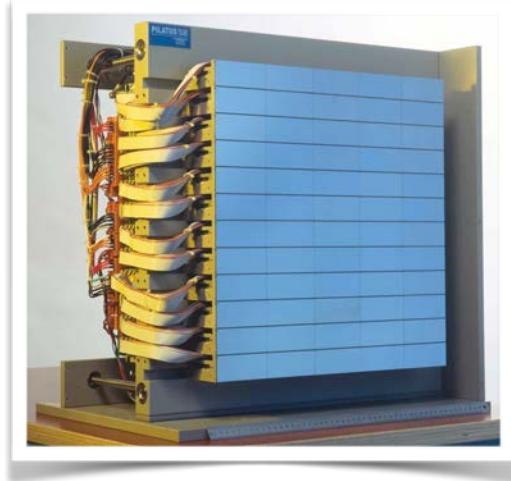
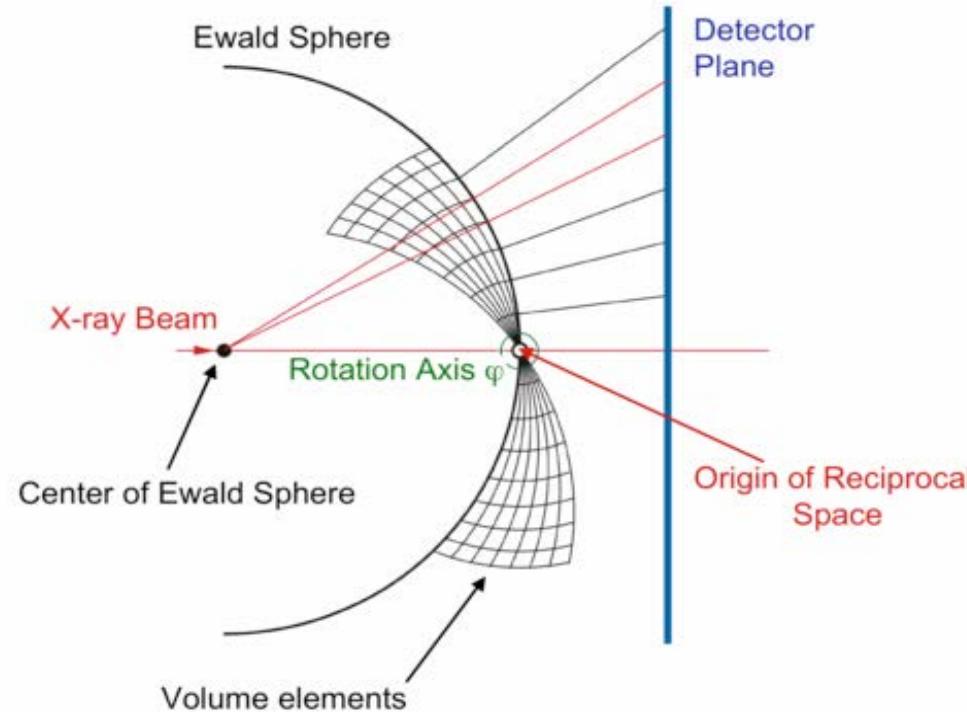
Incident Beam



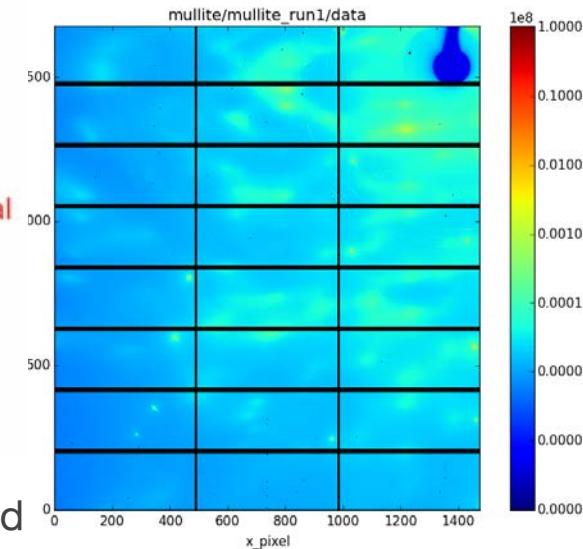
Cryocooler

Sample

Measuring X-ray Diffuse Scattering with Continuous Rotation Method



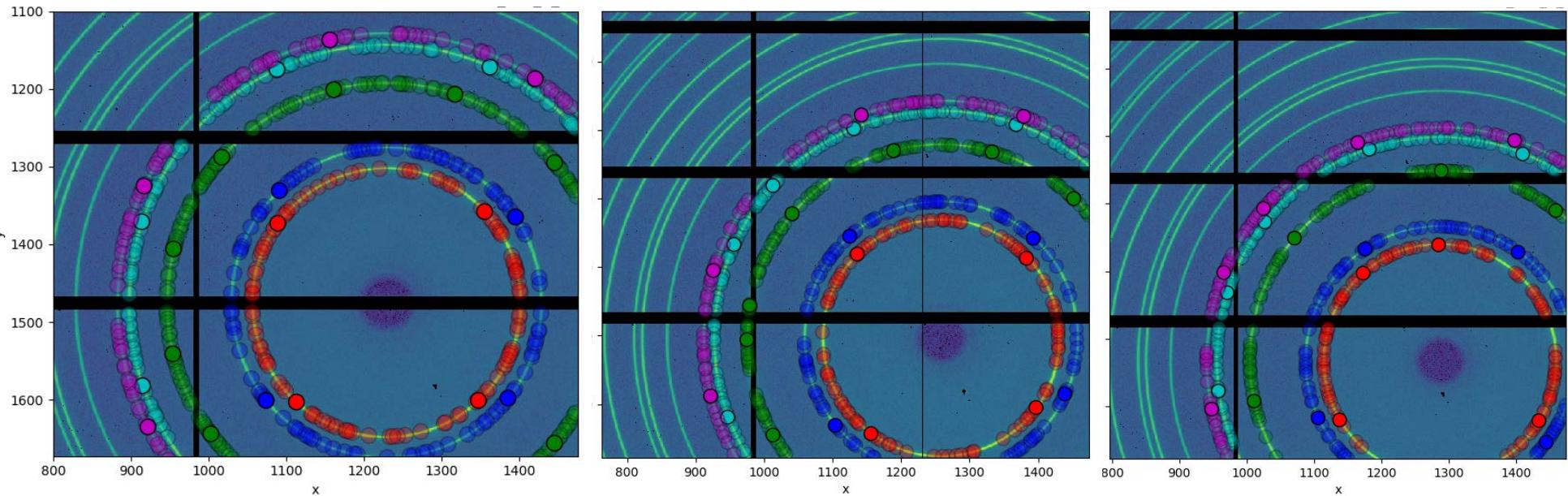
Pilatus 2M Detector



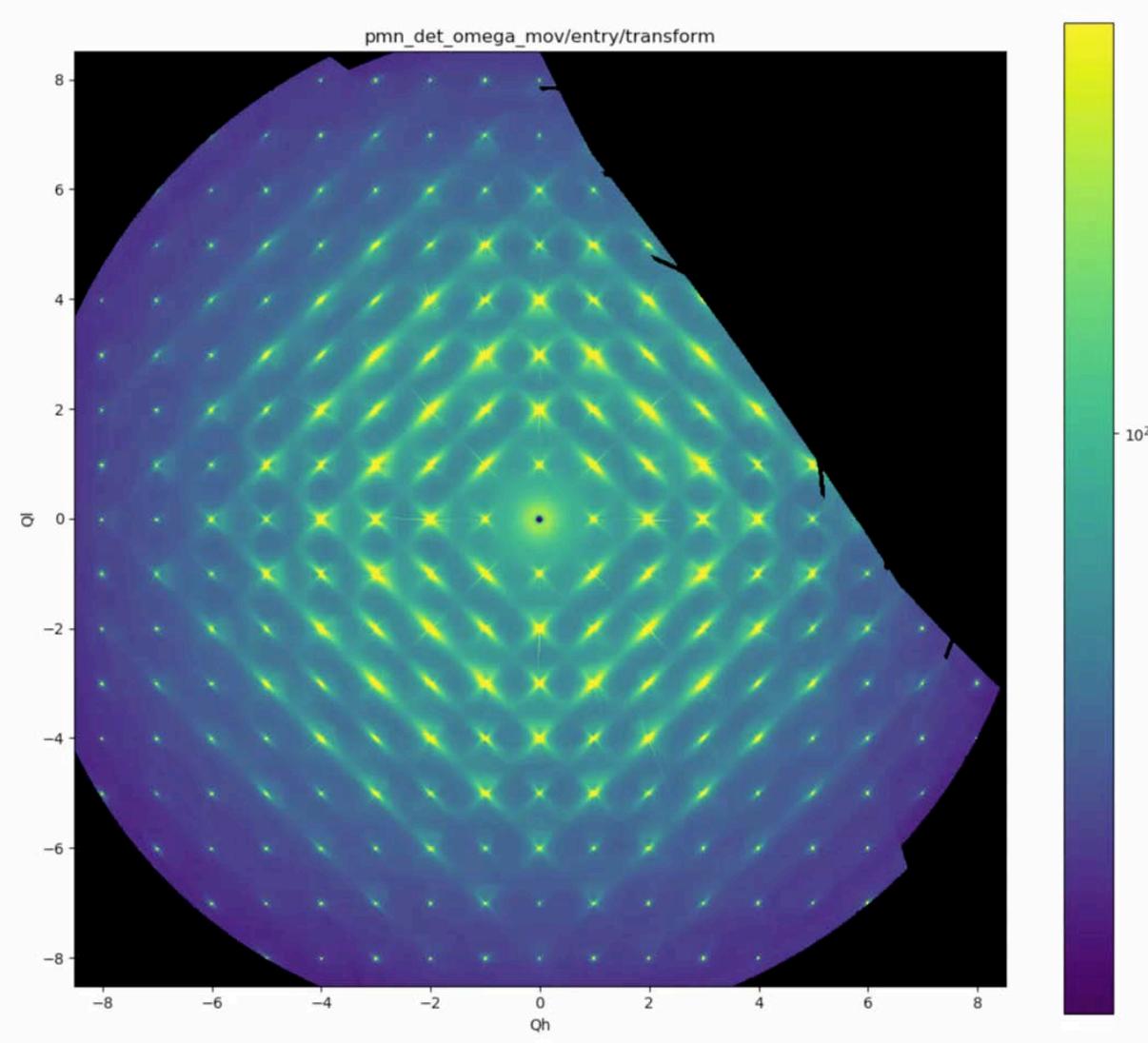
- The sample is continuously rotated in shutterless mode at 1° per second
- A fast area detector (e.g., a Pilatus 2M) acquires images at 10 frames per second
 - i.e., 3600 x 8MB frames ~ 30GB every 6 minutes
- The detector needs low background, high dynamic range, and energy discrimination
 - Ideally, this is performed with high-energy x-rays, e.g., 80 to 100 keV

Experiment Workflow

- ▶ Powder calibration to determine detector distance, centers, and tilts
- ▶ Bragg peak search to optimize the sample/detector geometry
- ▶ Determine the orientation matrix
- ▶ Perform coordinate transformation at each detector position
- ▶ Merge the three transforms

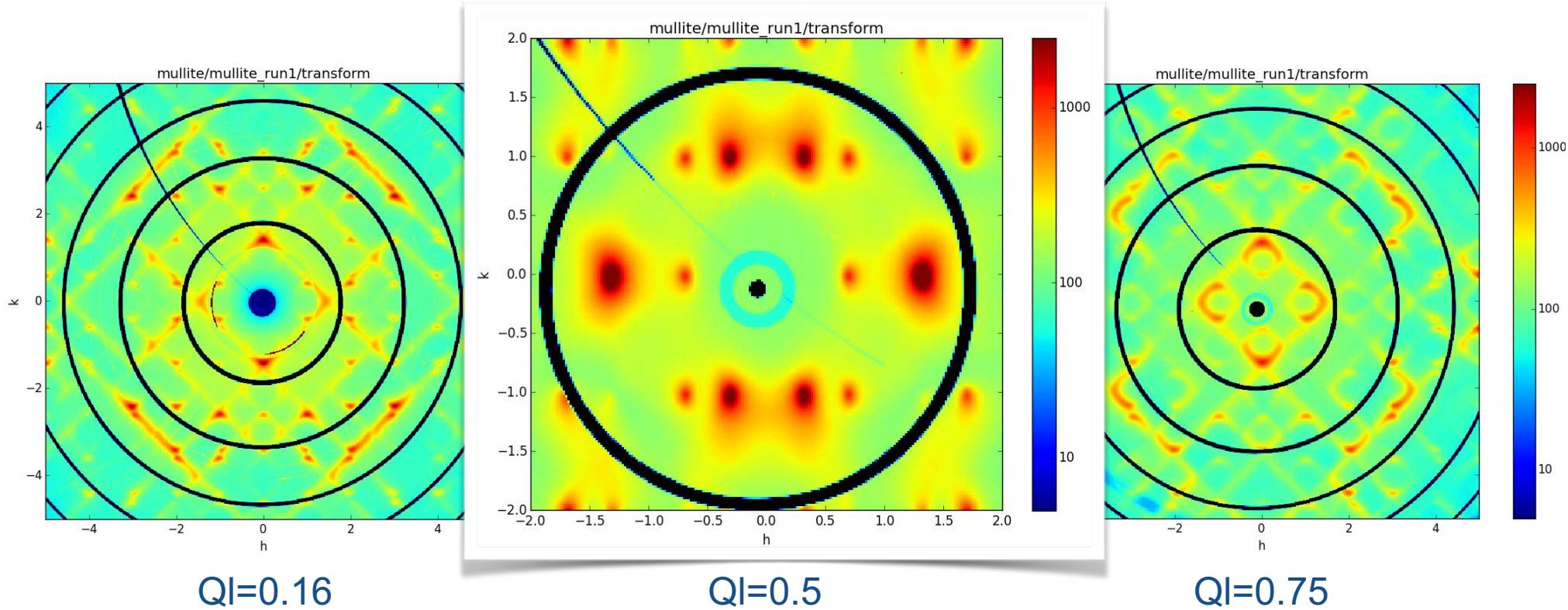


Diffuse Scattering in the Relaxor $\text{PbMg}_{1/3}\text{Nb}_{2/3}\text{O}_3$



3D Diffuse Scattering in Mullite

- There is strong diffuse scattering throughout reciprocal space
- The shape of the diffuse scattering is strongly dependent on the value of QI
- There are incipient superlattice peaks at $\mathbf{Q} = 0.5 \mathbf{c}^* + 0.31 \mathbf{a}^*$



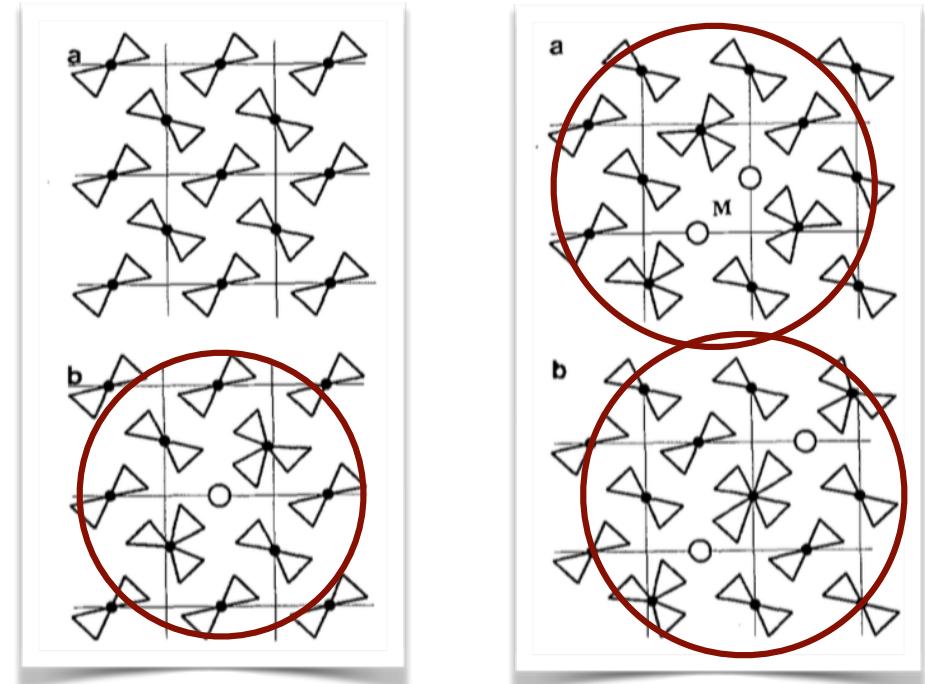
Monte Carlo Analysis

- In a classic analysis, Richard Welberry and colleagues developed a set of interaction energies to model mullite disorder
- Interaction energies were initialized:
 - insights from chemical intuition
 - insights from the measured diffuse scattering
- The diffuse scattering was calculated using a Monte Carlo algorithm to generate vacancy distributions first in 2D slices and then in 3D

$$P_i = \frac{e^{-V_i}}{1 + e^{-V_i}},$$

where,

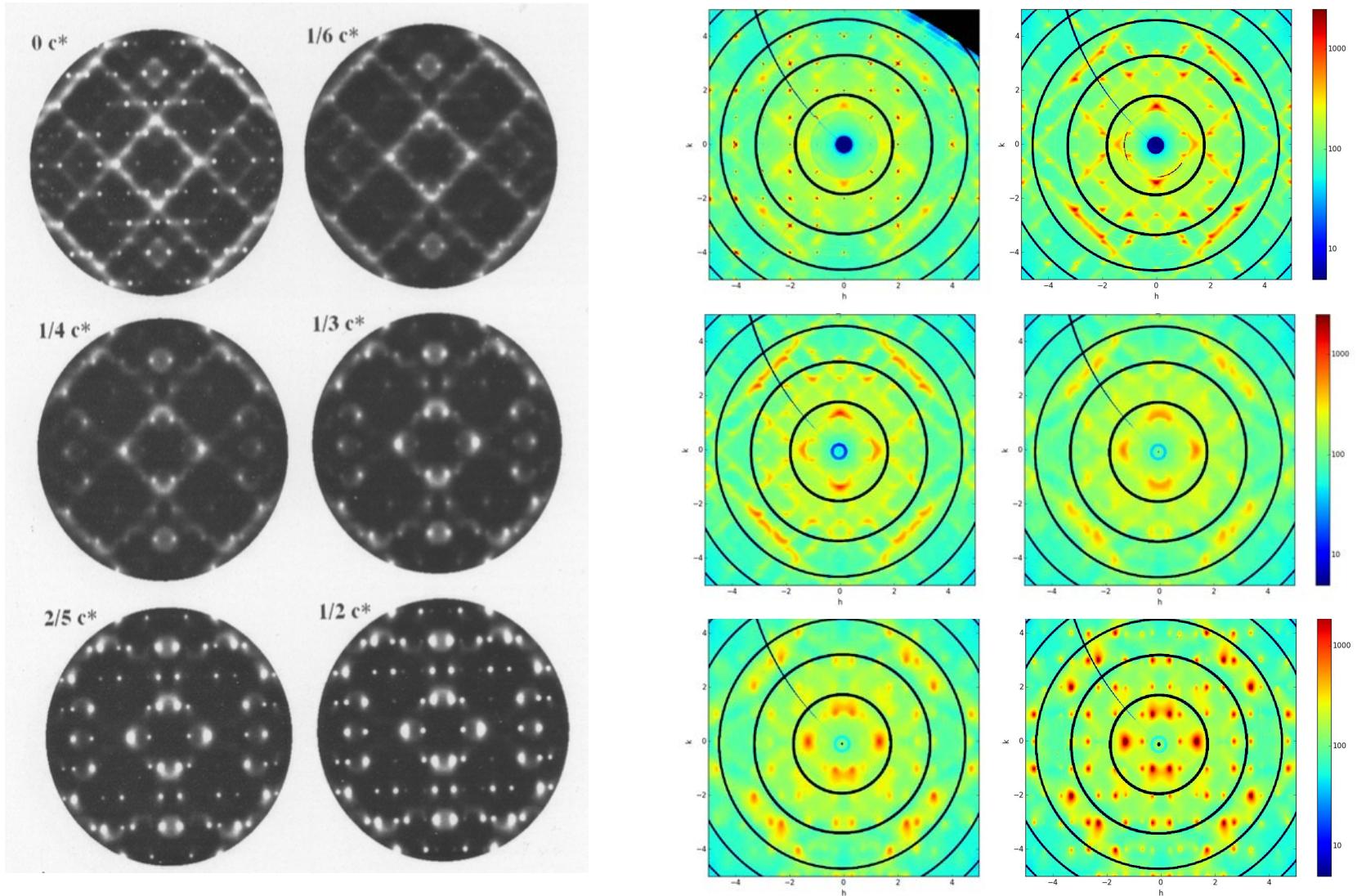
$$V_i = \frac{\sum_j E_{ij}}{kT} + \frac{(N_v - N_v^o)^2}{N_v^o} \operatorname{sgn}(N_v - N_v^o).$$



Interatomic vector	α_{lmn}	Interatomic vector	α_{lmn}
$\frac{1}{2}\langle 110 \rangle$	-0.24	$\langle 020 \rangle$	+0.13
$[110]$	-0.23	$\frac{1}{2}\langle 310 \rangle$	+0.22
$[1-10]$	-0.05	$\frac{1}{2}\langle 130 \rangle$	-0.01
$\langle 100 \rangle$	-0.06	$\langle 101 \rangle$	+0.07
$\langle 010 \rangle$	+0.22	$\langle 011 \rangle$	-0.12
$\langle 001 \rangle$	-0.03	$\frac{1}{2}\langle 330 \rangle$	+0.17
$\frac{1}{2}[1-12]$	+0.12	$\langle 111 \rangle$	-0.01
$\frac{1}{2}[112]$	+0.12	$\frac{1}{2}\langle 312 \rangle$	-0.11
$\langle 200 \rangle$	-0.12	$\frac{1}{2}\langle 332 \rangle$	-0.07

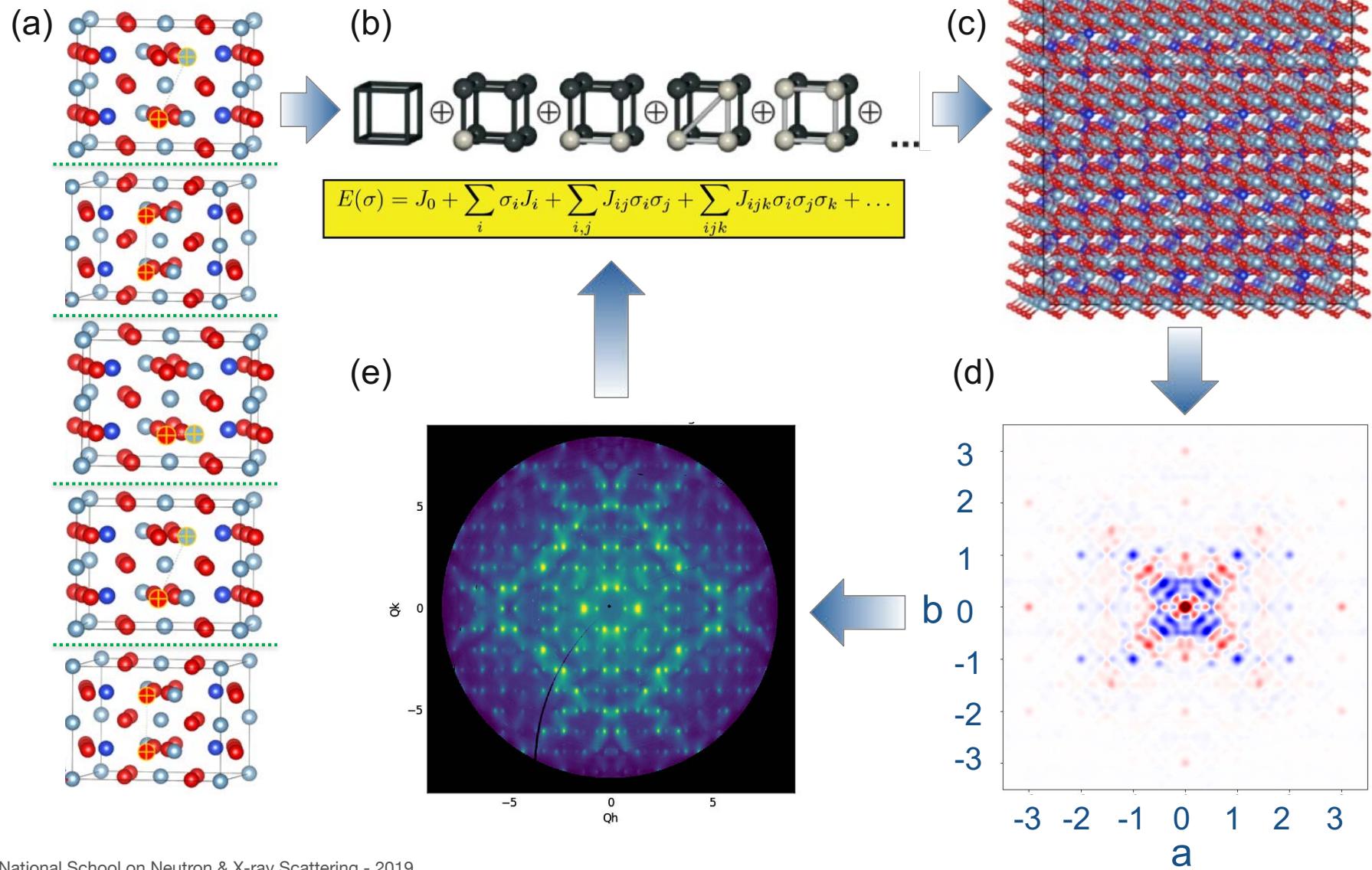
B. D. Butler, T. R. Welberry, & R. L. Withers, Phys Chem Minerals **20**, 323 (1993)

Monte Carlo Analysis Results

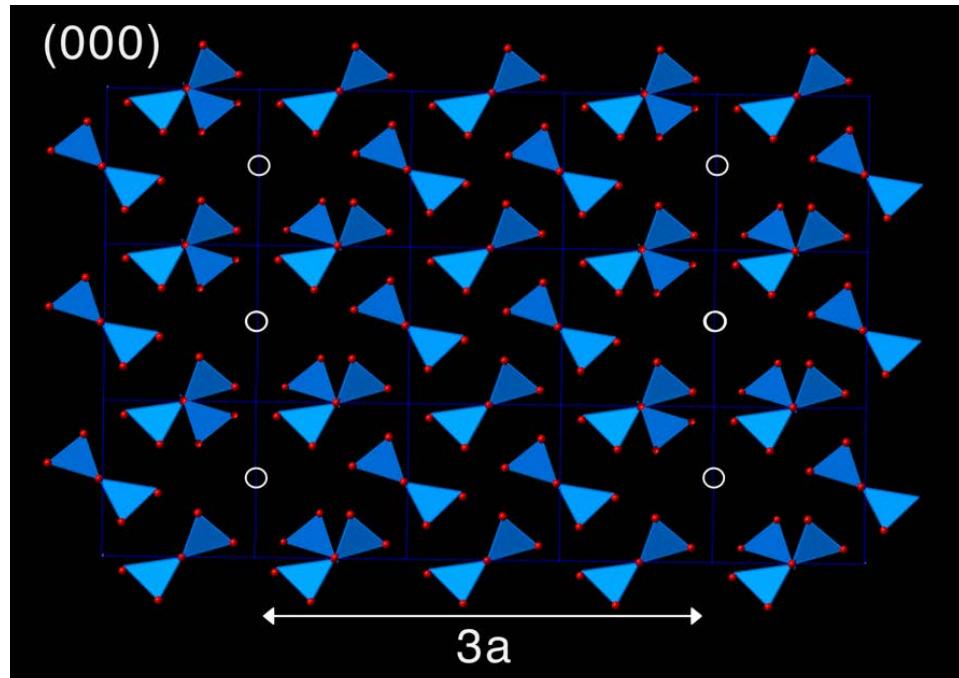


Vacancy Short-Range Order in Mullite

A First-Principles Approach (ab initio HRMC)

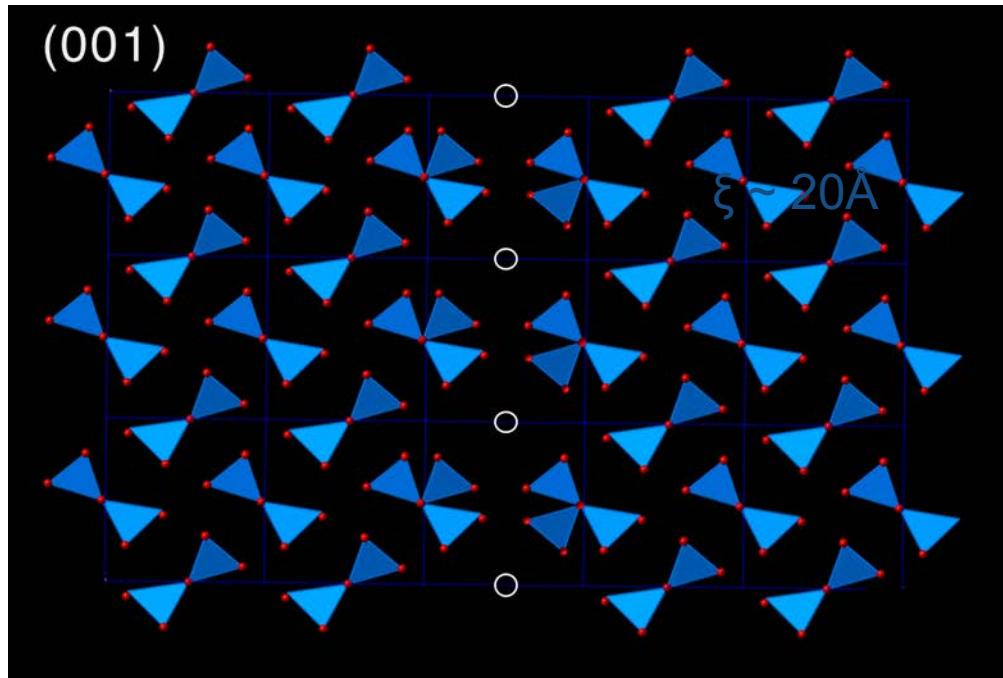


Nearly-Commensurate Vacancy Stripes in Mullite



$$\mathbf{c} = 0$$

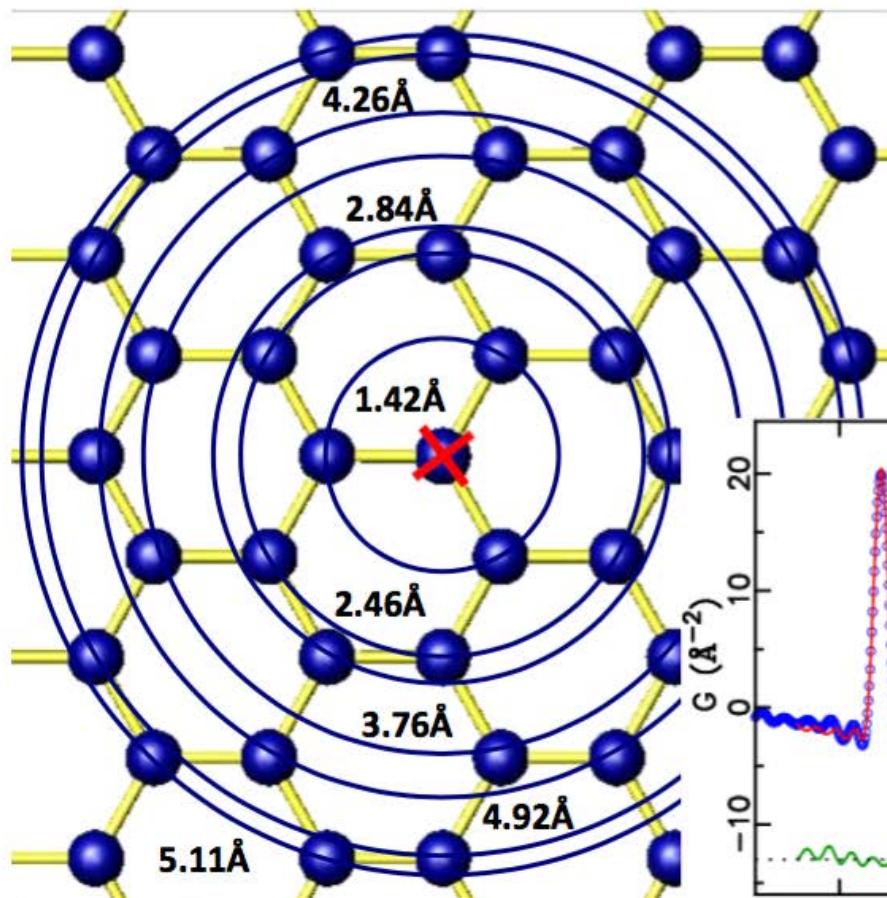
$$\mathbf{q} = \pm \frac{1}{2}\mathbf{c}^* \pm \frac{1}{3}\mathbf{a}^*$$



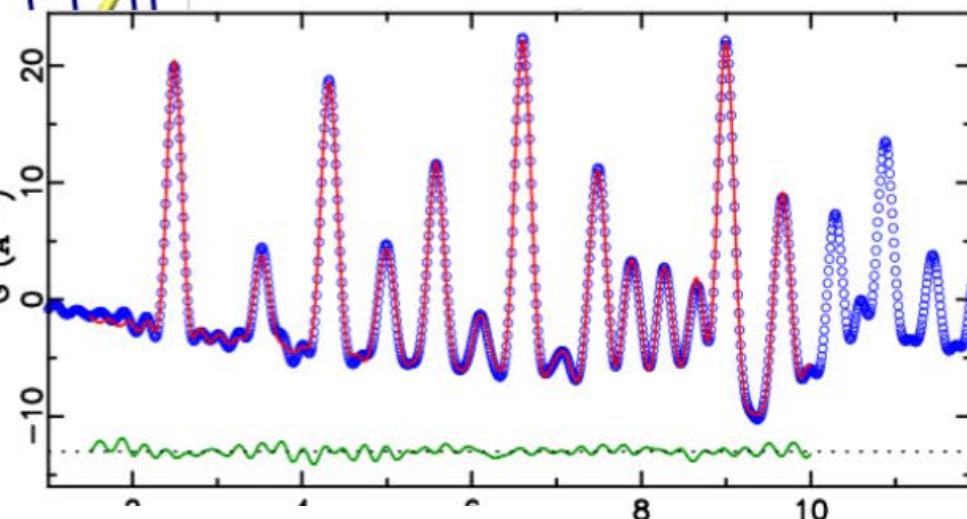
$$\mathbf{c} = 1.0$$

Case Study 2: Sodium-Intercalated V₂O₅ 3D- Δ PDF

Pair Distribution Function Analysis



Radial atomic pair distribution function (PDF) gives the interatomic distance distribution, or “probability” of finding atomic pairs distance r apart



$$G(r) = 4\pi r[\rho(r) - \rho_o] = (2/\pi) \int_{Q=Q_{\min}}^{Q_{\max}} Q[S(Q)-1]\sin(Qr)dQ$$

Emil Bozin (ADD 2013)

Three-Dimensional Pair Distribution Functions

- ▶ The ability to measure three-dimensional $S(Q)$ over a wide range of reciprocal space provides the 3D analog of PDF measurements.
 - Total PDFs if Bragg peaks and diffuse scattering can be measured simultaneously
 - Δ -PDFs if the Bragg peaks are eliminated
 - using the punch and fill method
- ▶ This would allow a model-independent view of the measurements in real space.

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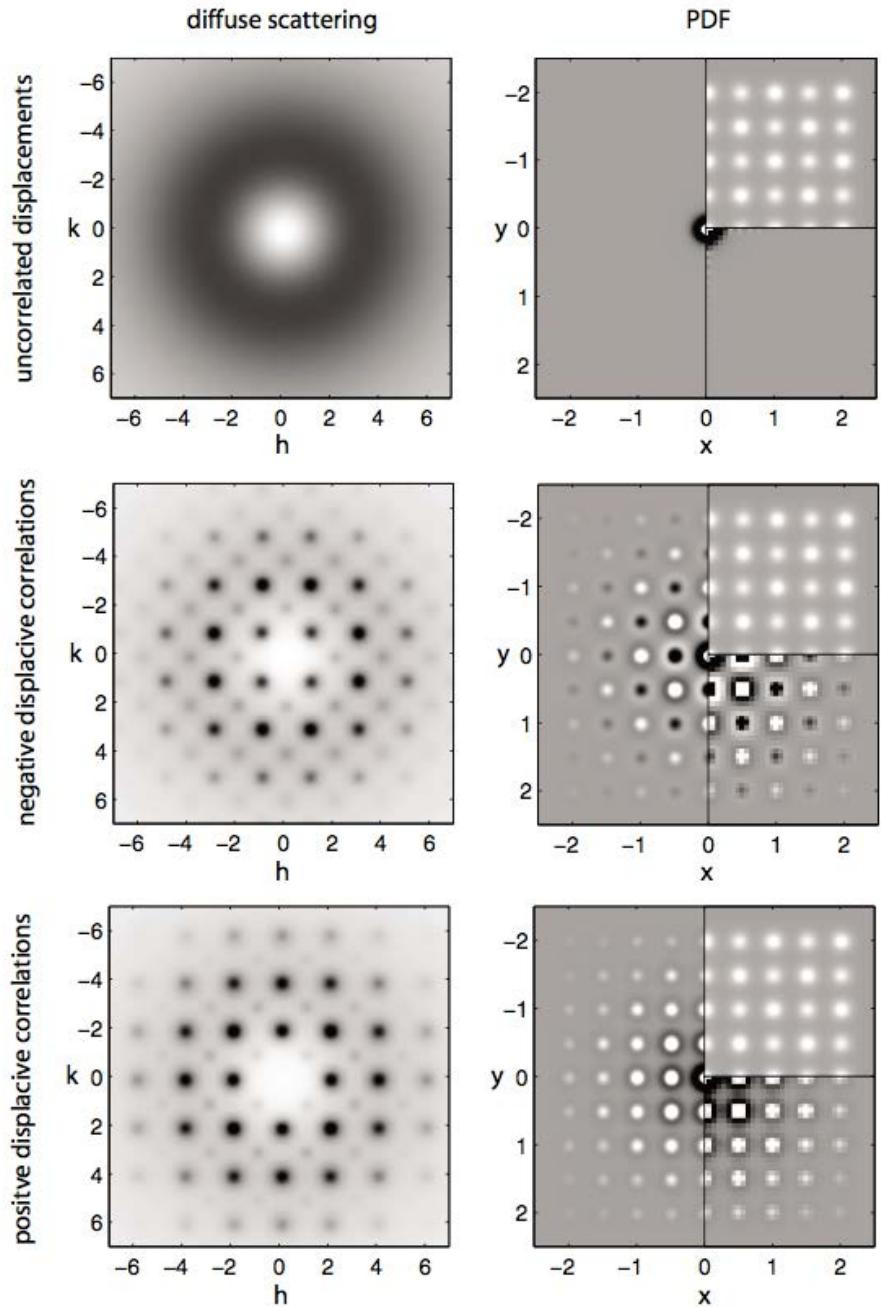
Z. Kristallogr. 2012, 227, 238–247 / DOI 10.1524/zkri.2012.1504

© by Oldenbourg Wissenschaftsverlag, München

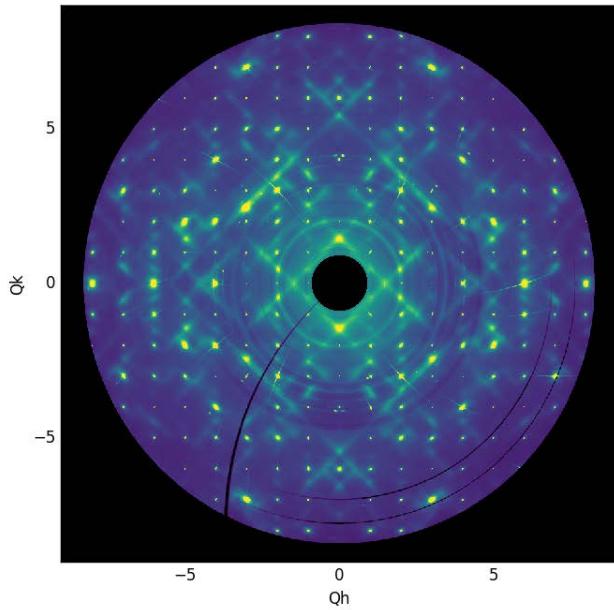
The three-dimensional pair distribution function analysis of disordered single crystals: basic concepts

Thomas Weber* and Arkadiy Simonov

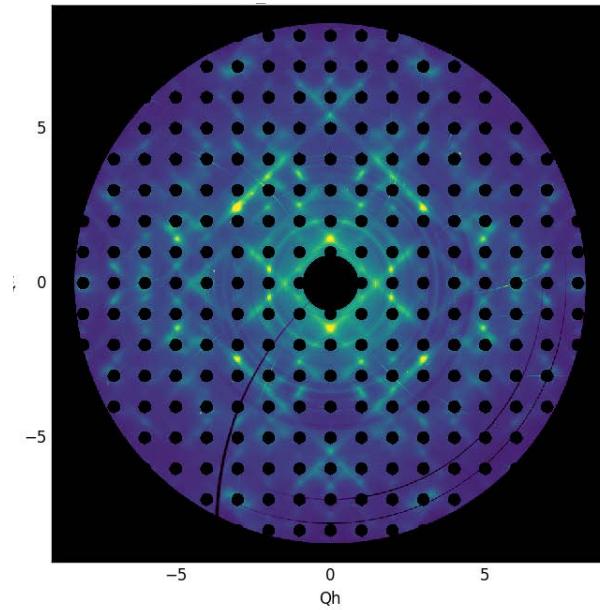
Laboratory of Crystallography, ETH Zurich Wolfgang-Pauli-Str. 10, 8093 Zurich, Switzerland



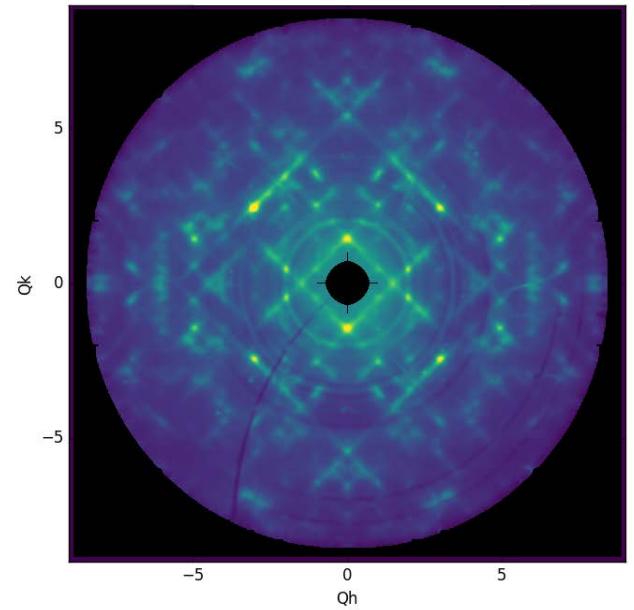
"Punch and Fill"



Symmetrize



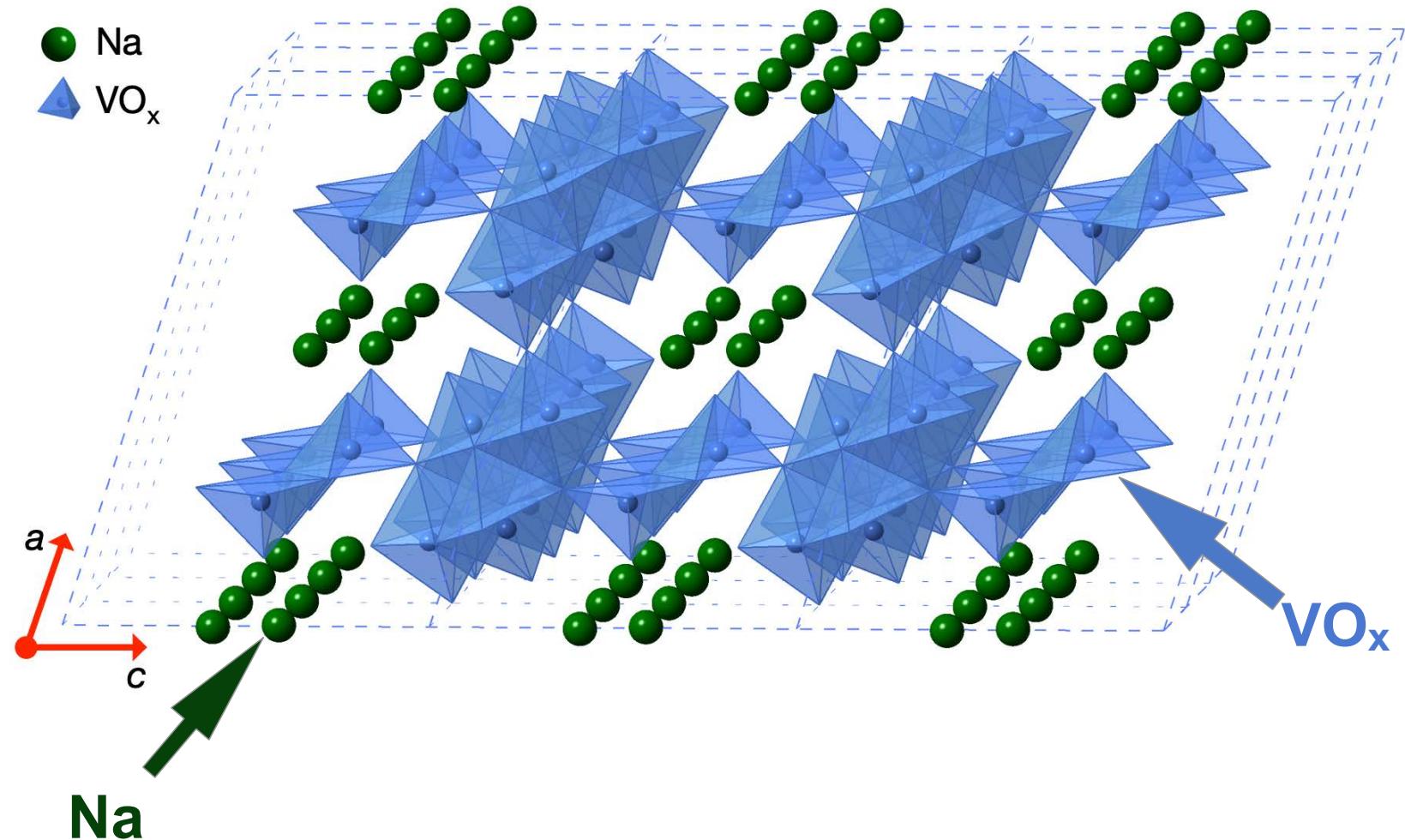
Punch



Fill

$$P_{\text{tot}}(\mathbf{r}) = FT[I(\mathbf{u})] = FT[|\bar{F}(\mathbf{u})|^2] + FT[|\Delta F(\mathbf{u})|^2] = P_{hkl}(\mathbf{r}) + \Delta P(\mathbf{r})$$

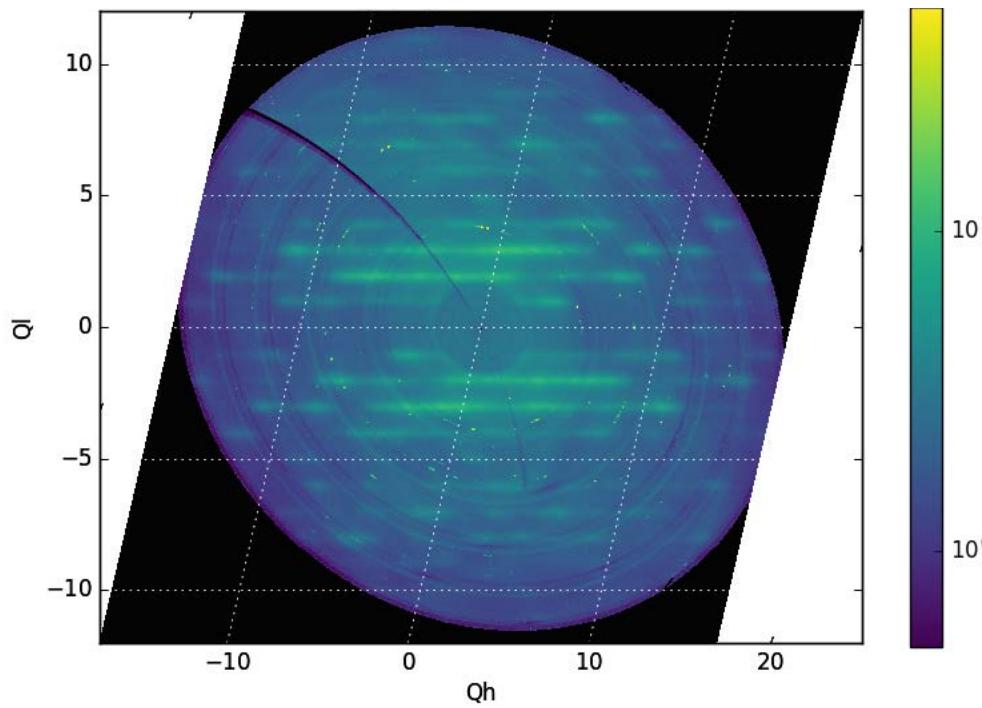
Sodium-Intercalated V_2O_5



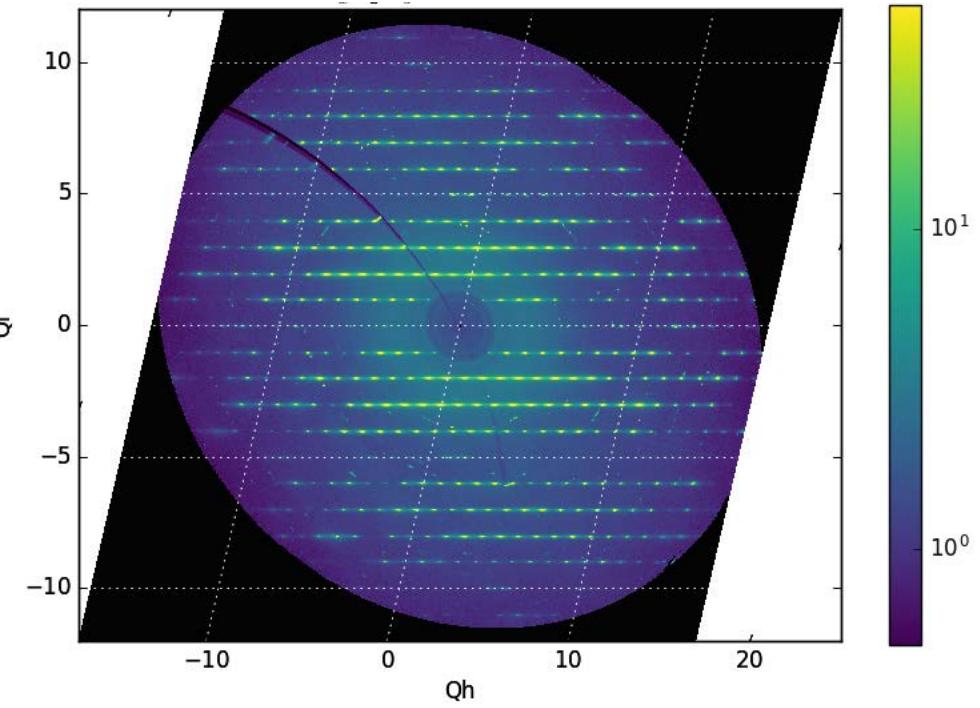
M. J. Krogstad, S. Rosenkranz, J. M. Wozniak, G. Jennings, J. P. C. Ruff, J. T. Vaughey, and R. Osborn
arXiv 1902.03318 (submitted to Nature Materials).

Diffuse Scattering in $\text{Na}_{0.2}\text{V}_2\text{O}_5$ and $\text{Na}_{0.4}\text{V}_2\text{O}_5$

$Qk=0.5$

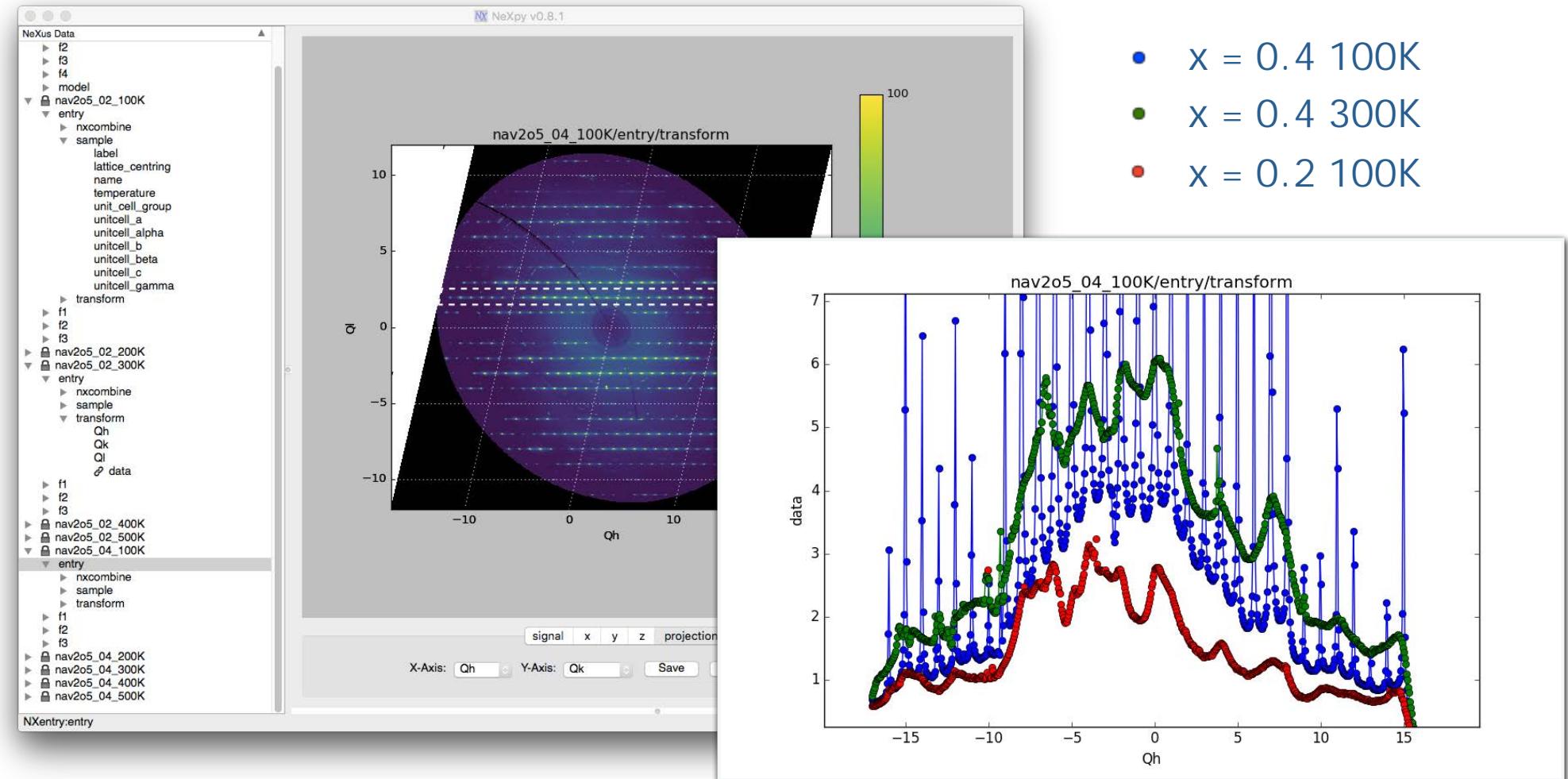


$\text{Na}_{0.4}\text{V}_2\text{O}_5$
300K

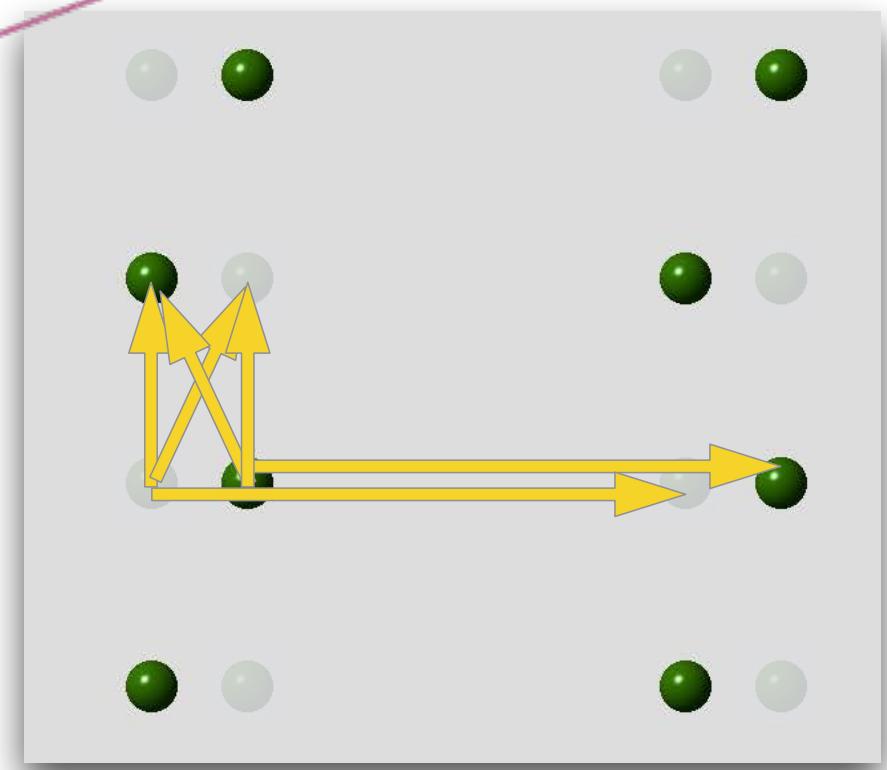
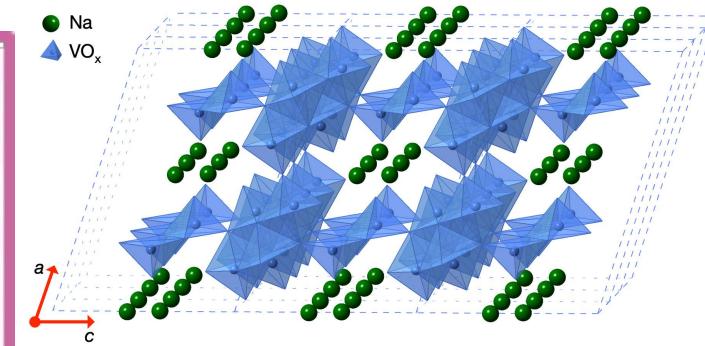
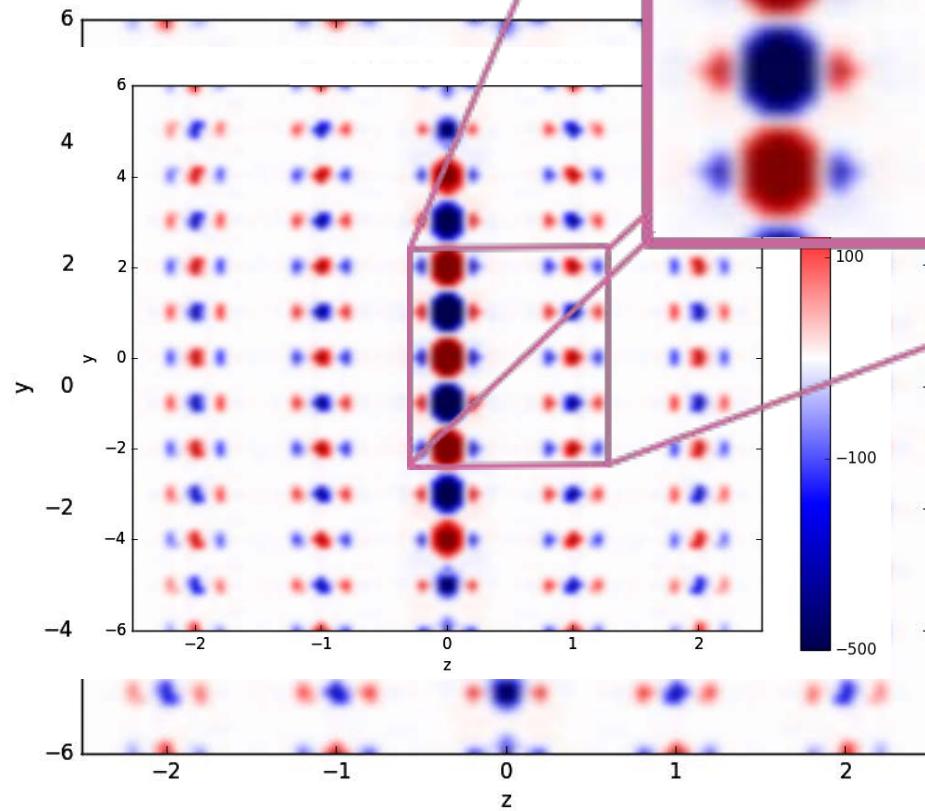


$\text{Na}_{0.4}\text{V}_2\text{O}_5$
100K

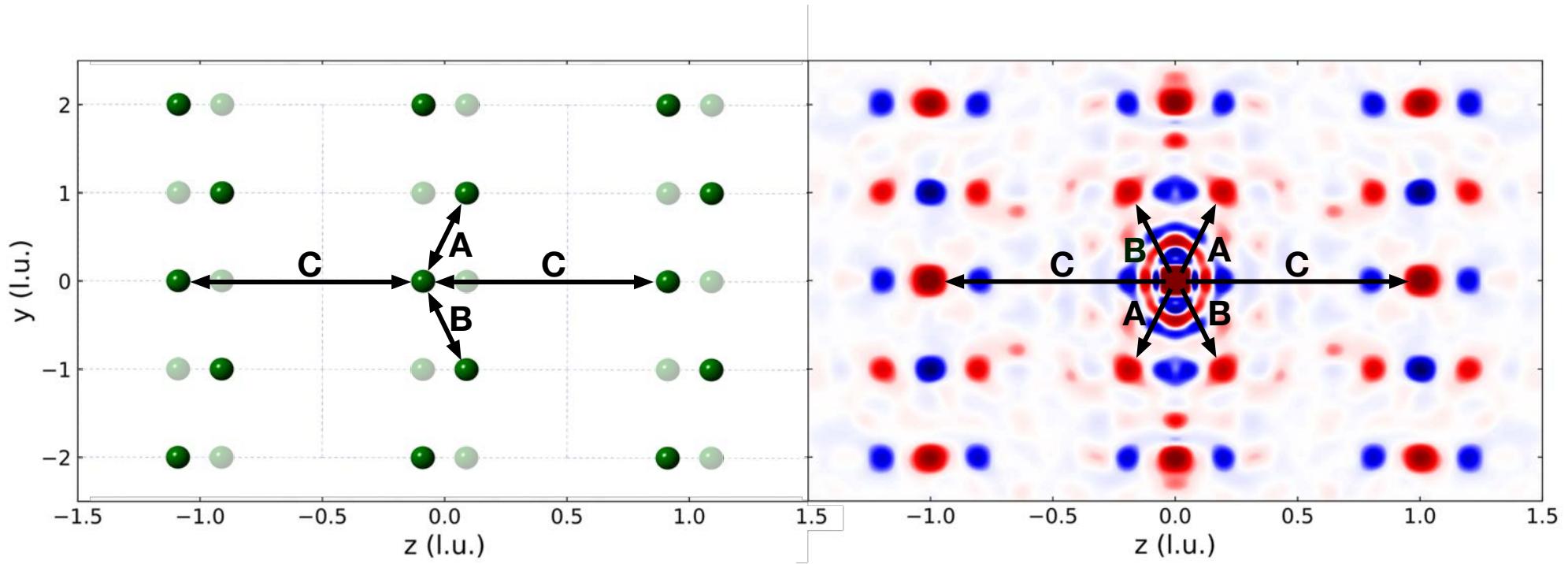
Sublattice Melting



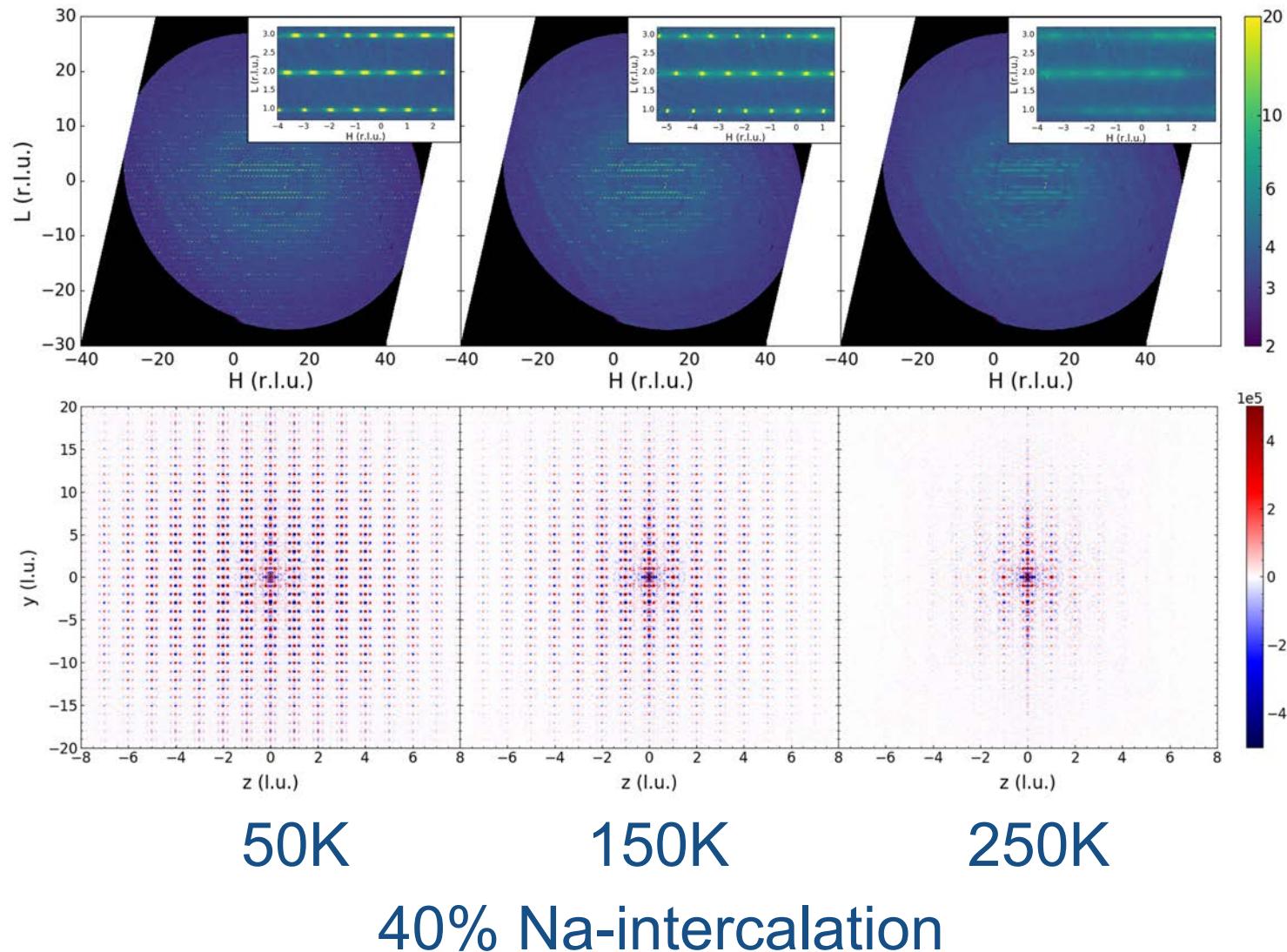
3D- Δ PDF Analysis of



Real Space vs 3D- Δ PDF

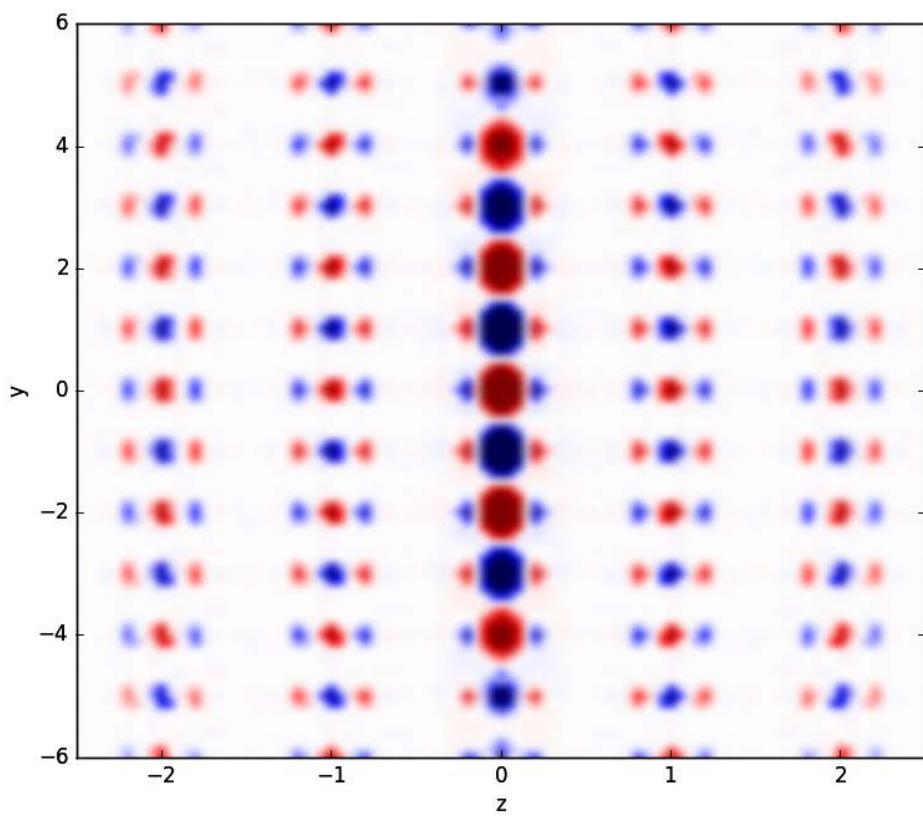
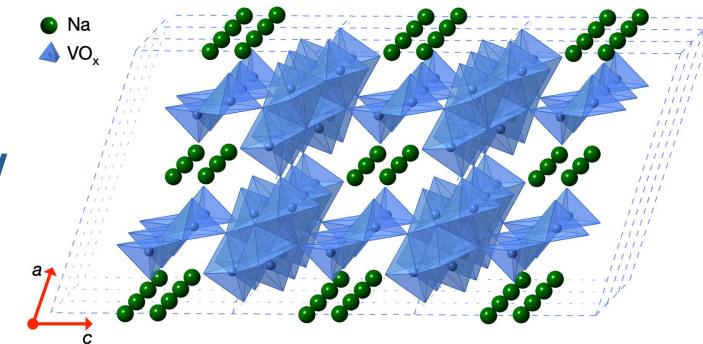


Order-Disorder Transition Viewed in Reciprocal and Real Space

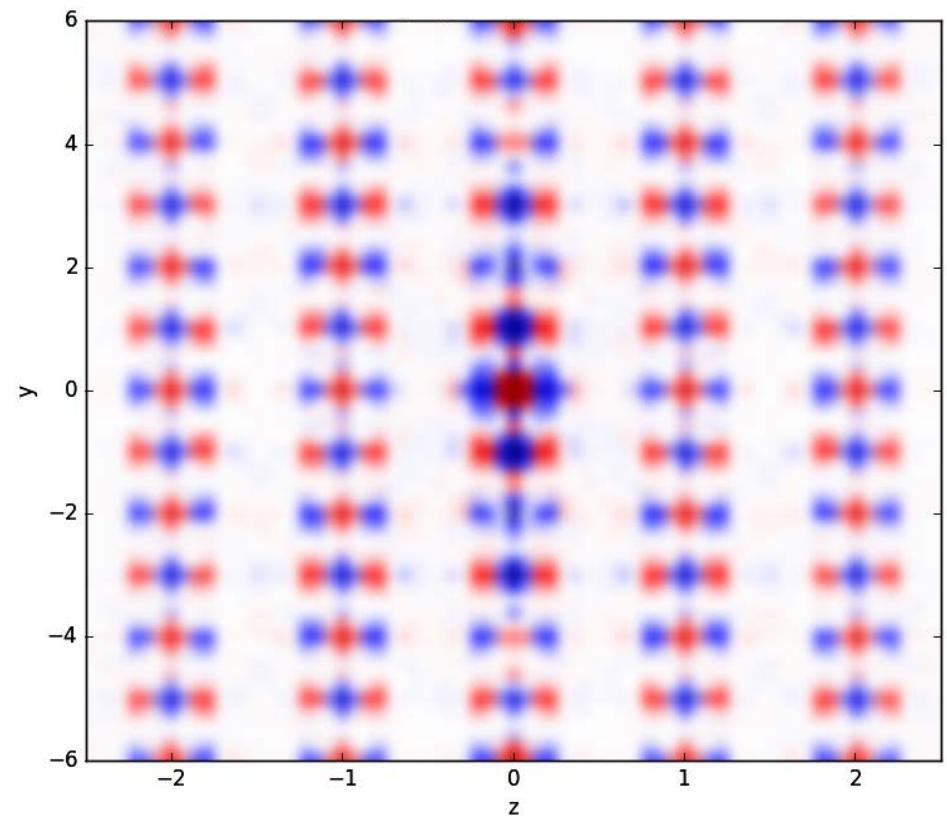


3D- Δ PDF: Importance of High Energy

- Expanding the Q-range enhances the real-space resolution

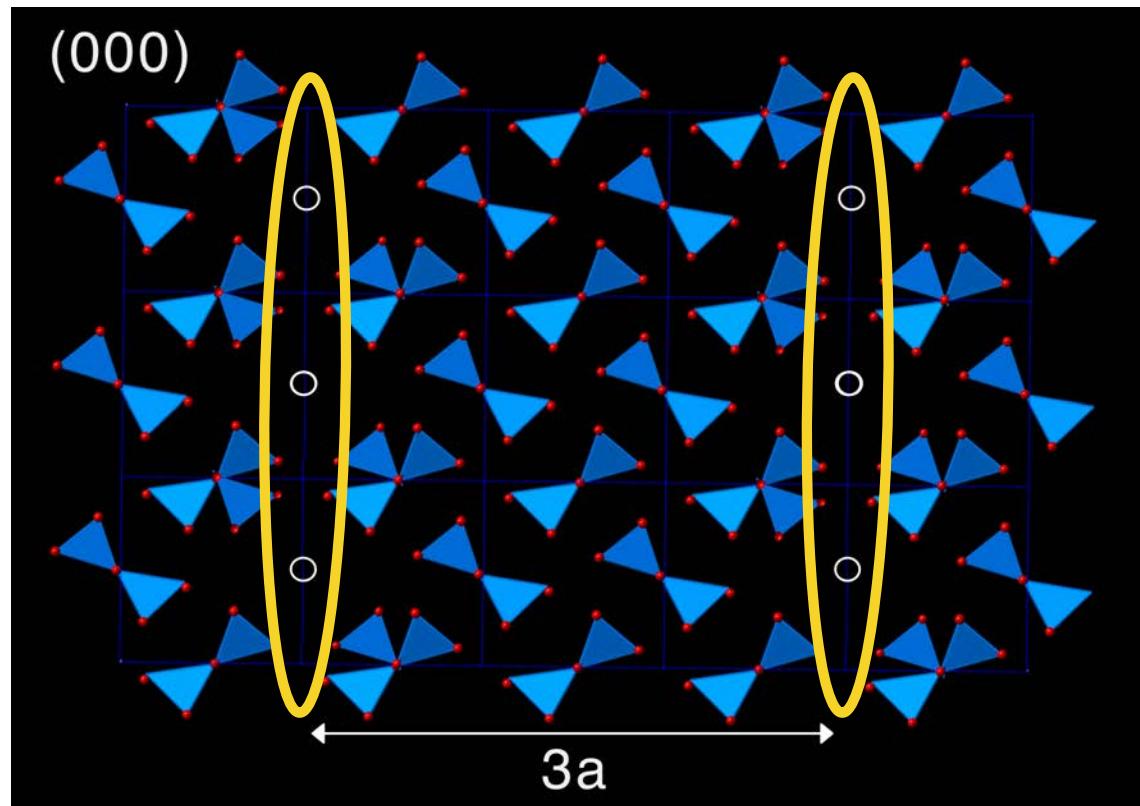
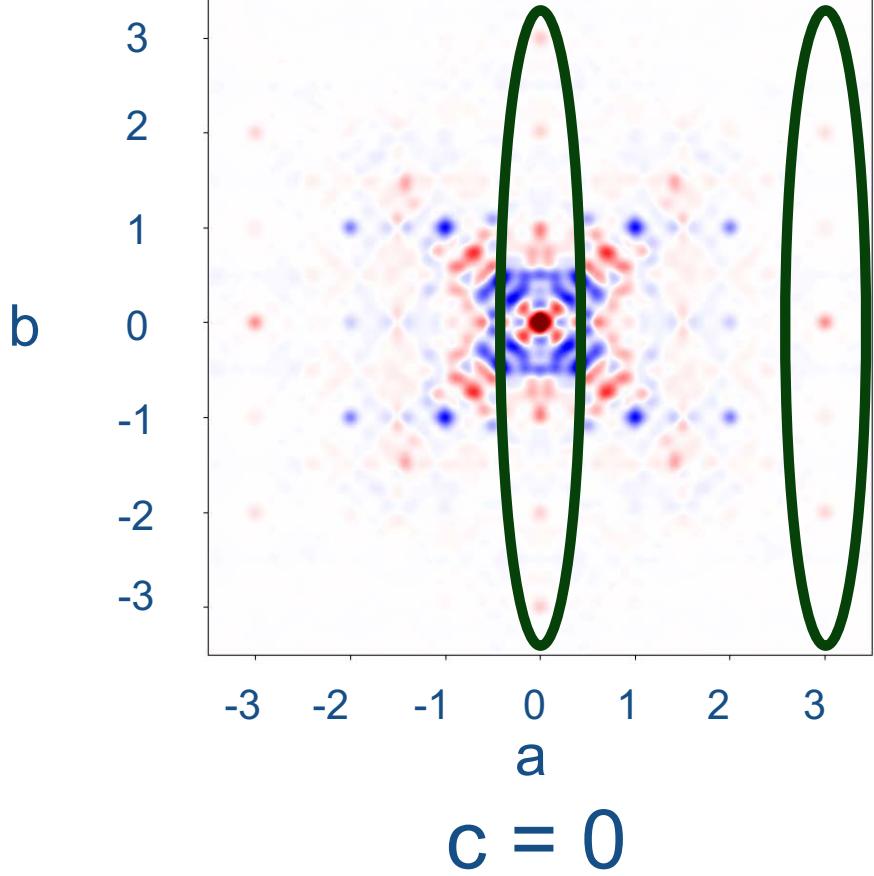


CHESS – A2 – 55keV



APS – 11-ID-D – 18.5keV

Back to Mullite



How do I look at static disorder?

Comparison of Elastic Scattering and the Static Approximation

$$\left(\frac{d^2\sigma}{d\Omega dE'} \right)_{coh} = b_{coh}^2 \frac{k'}{k} N \frac{1}{2\pi\hbar} \int G(\vec{r}, t) e^{i(\vec{Q} \cdot \vec{r} - \omega t)} d\vec{r} dt$$

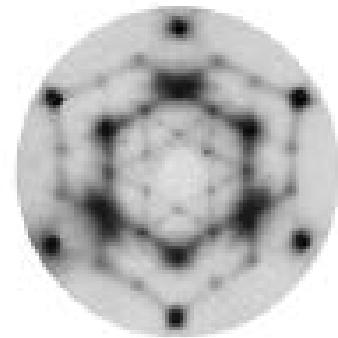
where $G(\vec{r}, t) = \frac{1}{(2\pi)^3} \frac{1}{N} \int e^{-i\vec{Q} \cdot \vec{r}} \sum_{i,i'} \langle e^{-i\vec{Q} \cdot \vec{R}_{j'}(0)} e^{i\vec{Q} \cdot \vec{R}_j(t)} \rangle$

$$\left(\frac{d\sigma}{d\Omega} \right)_{coh}^{static} = b_{coh}^2 N \int G(\vec{r}, 0) e^{i\vec{Q} \cdot \vec{r}} d\vec{r} \quad \text{since } \hbar\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iat} d(\hbar\omega)$$

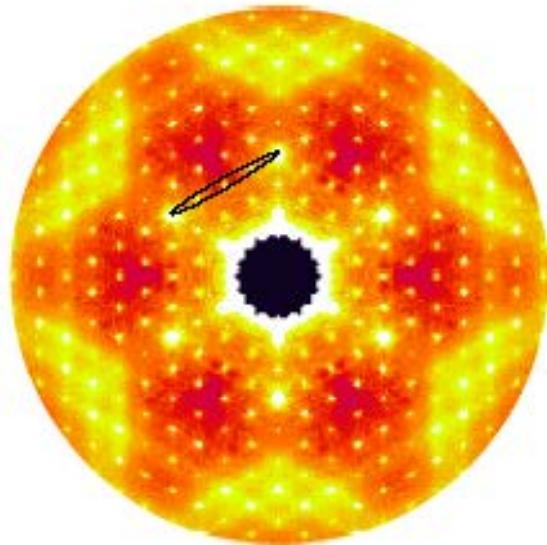
$$\left(\frac{d\sigma}{d\Omega} \right)_{coh}^{elastic} = b_{coh}^2 N \int G(\vec{r}, \infty) e^{i\vec{Q} \cdot \vec{r}} d\vec{r}$$

- Reference: Roger Pynn, National School of Neutron and X-ray Scattering, 2018

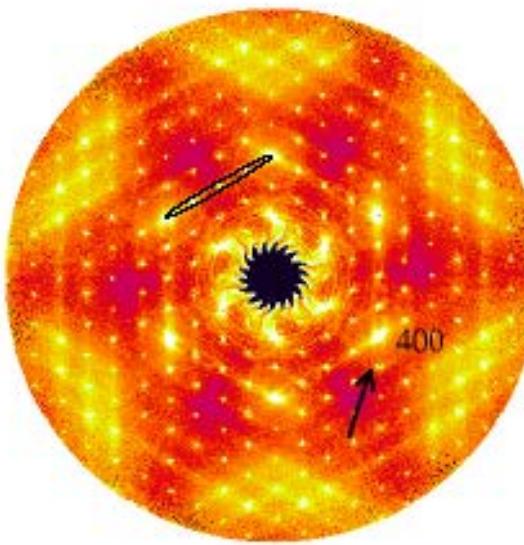
Importance of Elastic Discrimination



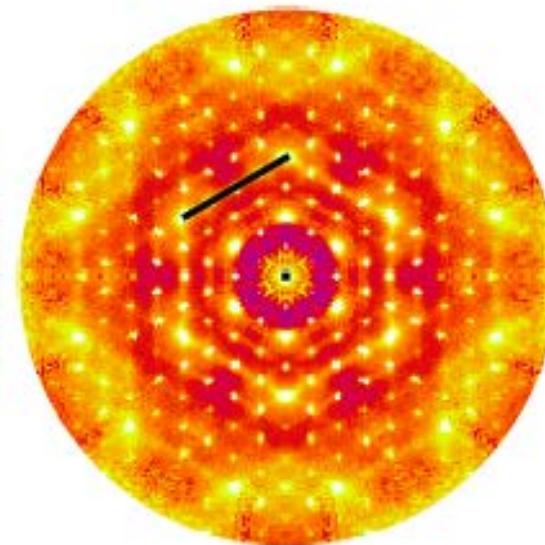
BENZIL



Detector 1
 $2\theta \sim 142.5^\circ$



Detector 2
 $2\theta \sim 90.0^\circ$



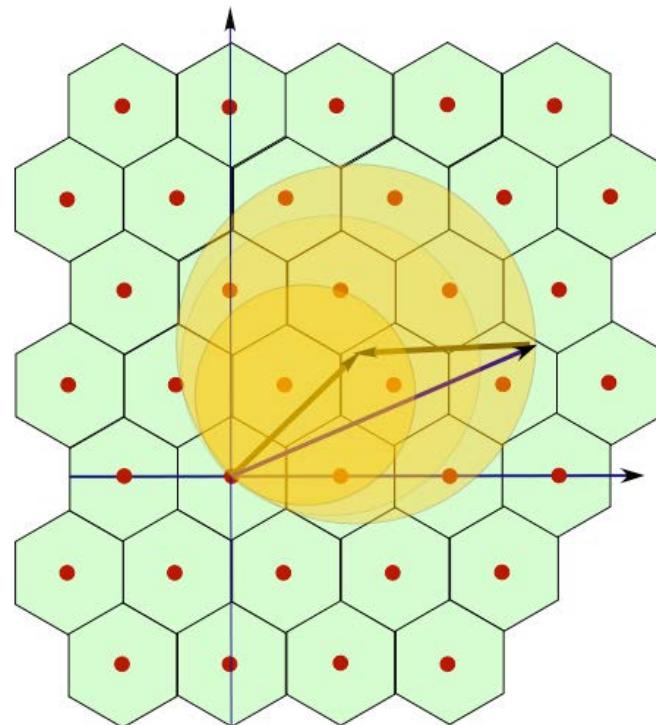
Detector 3
 $2\theta \sim 37.5^\circ$

$$Q = 4\pi \frac{\sin\theta}{\lambda}$$

Measuring Large Volumes of Reciprocal Space

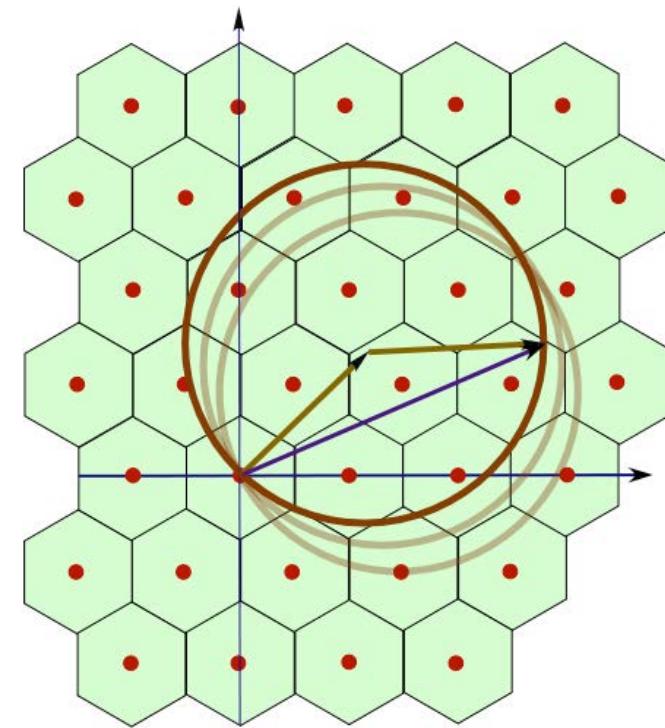
Conventional Time-of-Flight Neutron Methods

White Beam:
efficient



NO energy discrimination

Fixed k_i :
energy resolved



NOT efficient

Cross Correlation Chopper

TOF Laue Diffractometer

- highly efficient data collection
- wide dynamic range in Q

Statistical Chopper

- elastic energy discrimination
- optimum use of white beam

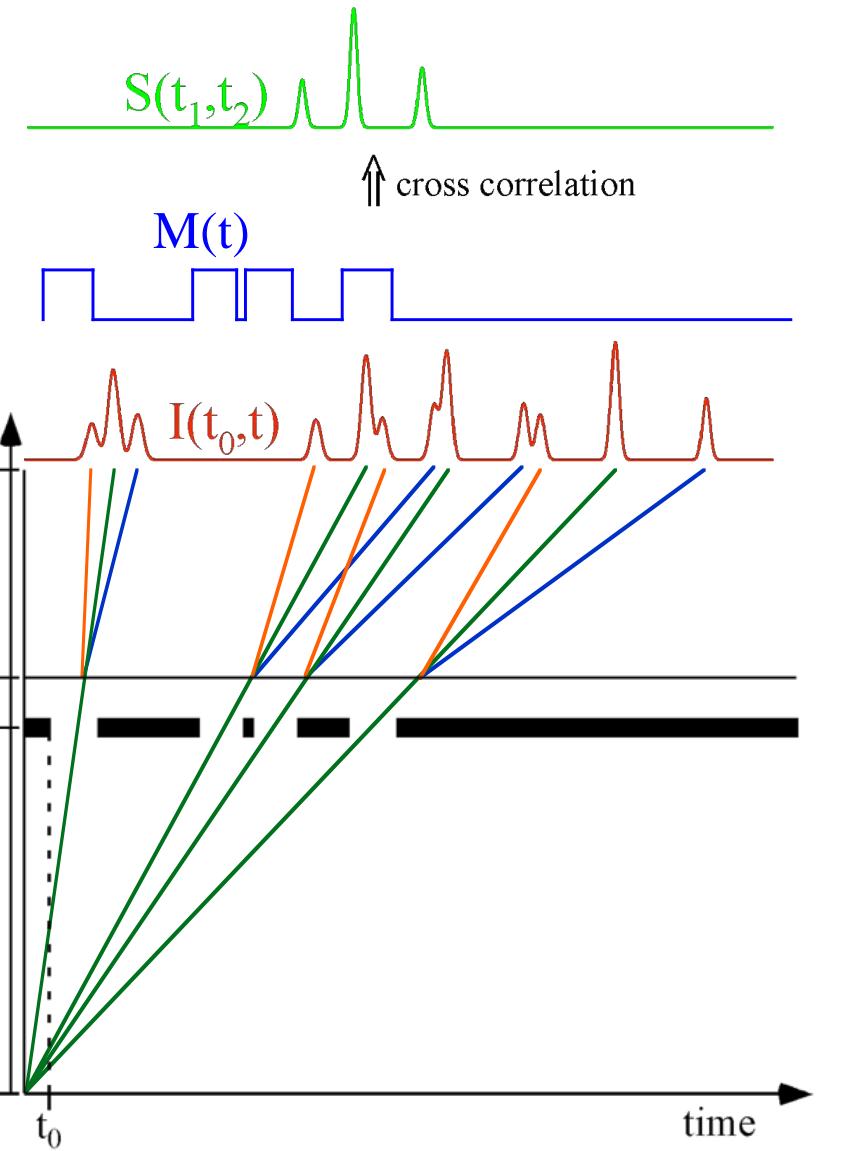
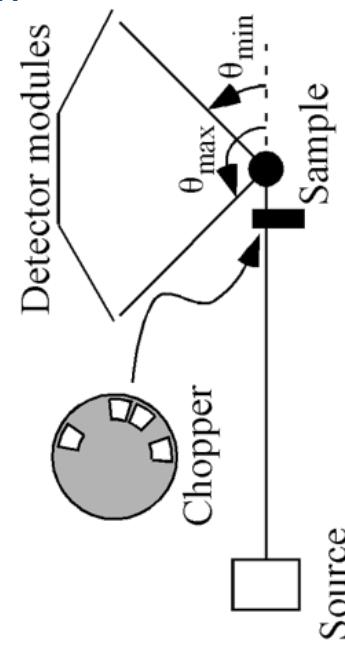
Sample with :
elastic scattering

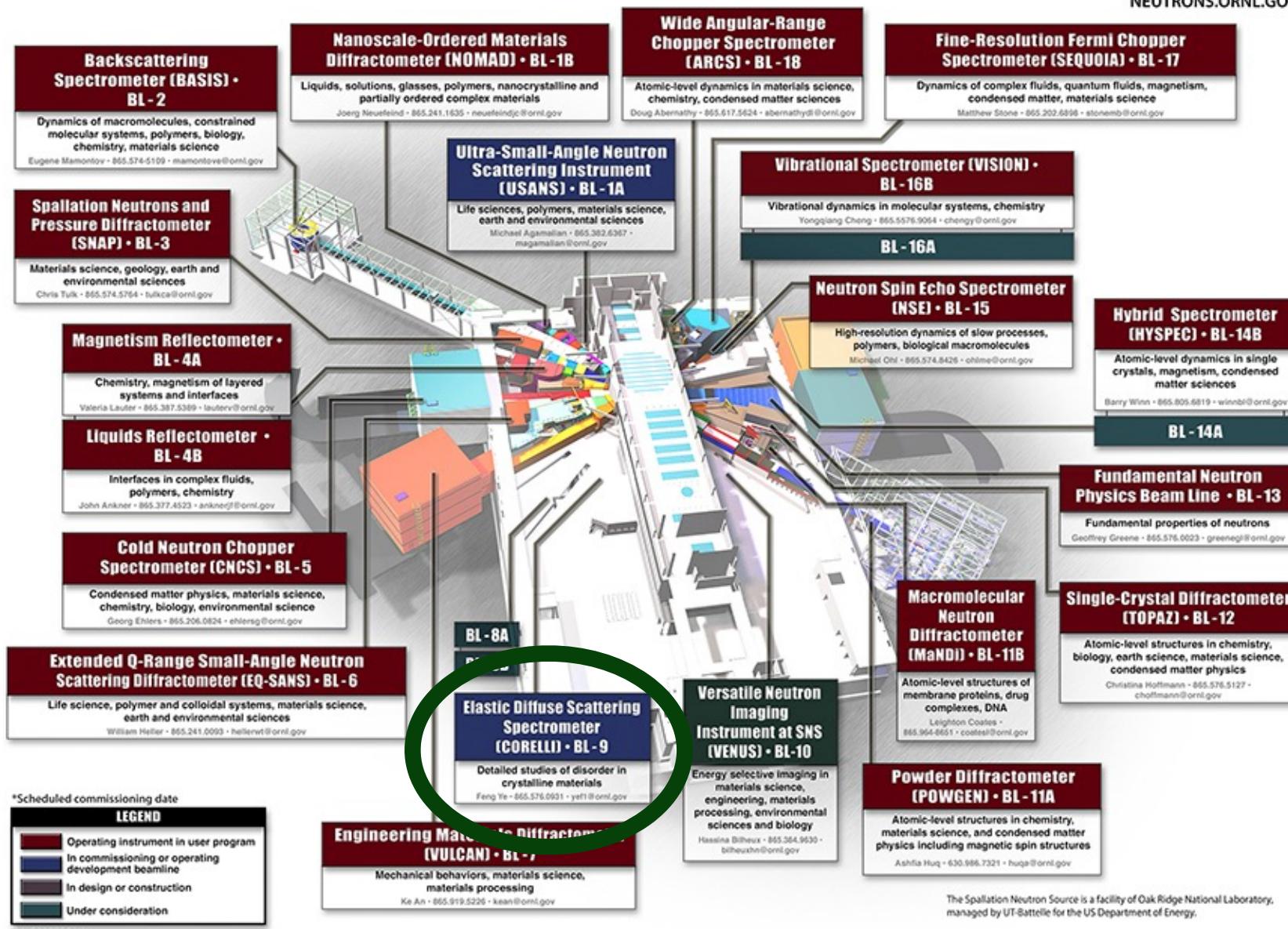
$$\hbar\omega = 0$$

inelastic excitations

$$\hbar\omega = +E_0$$

$$\hbar\omega = -E_0$$



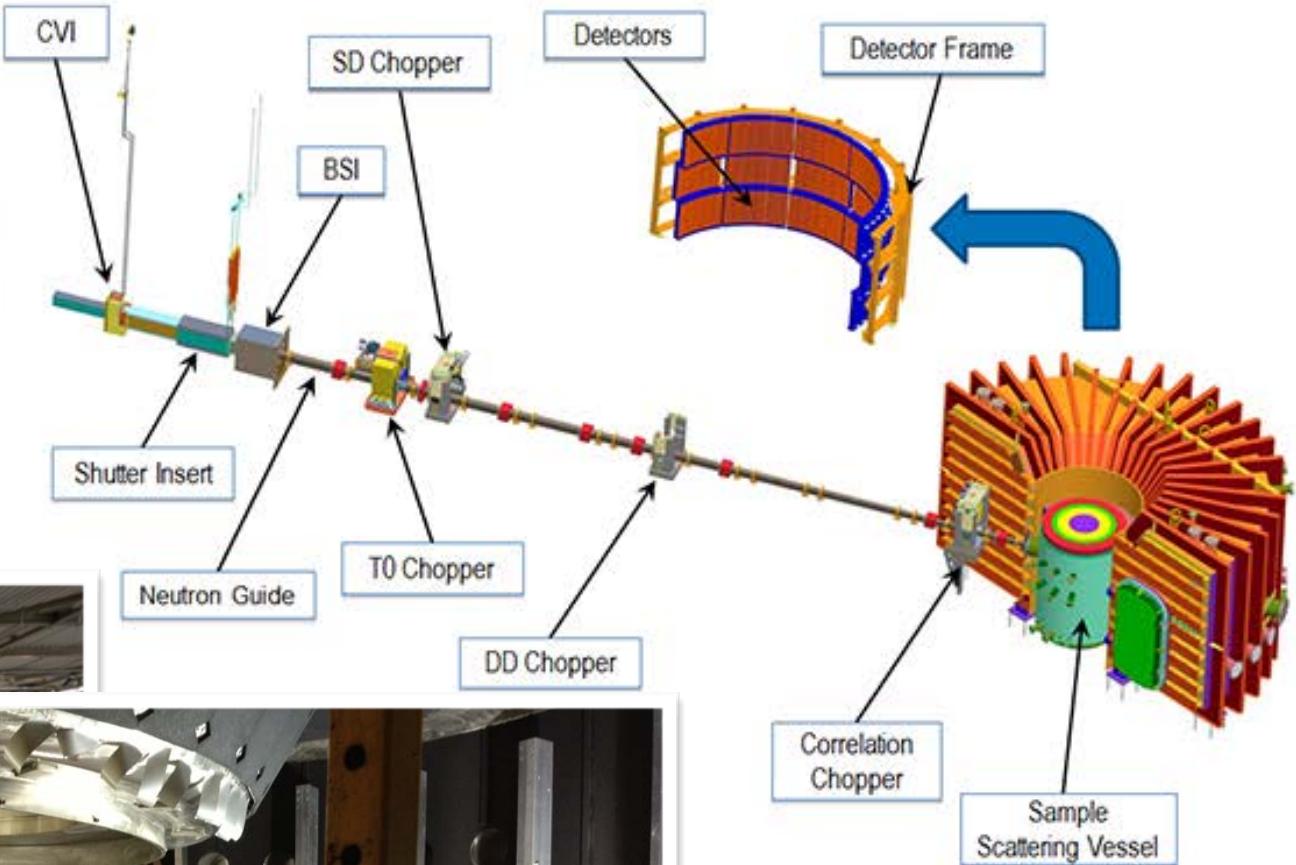


Corelli

Instrument Scientists

Feng Ye

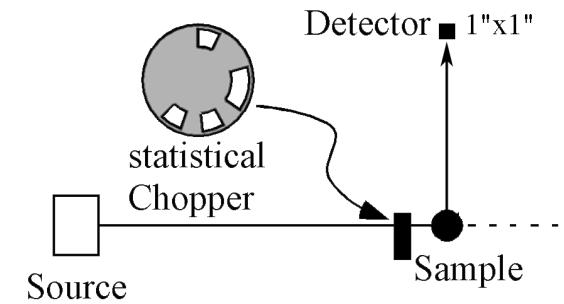
Yaohua Liu



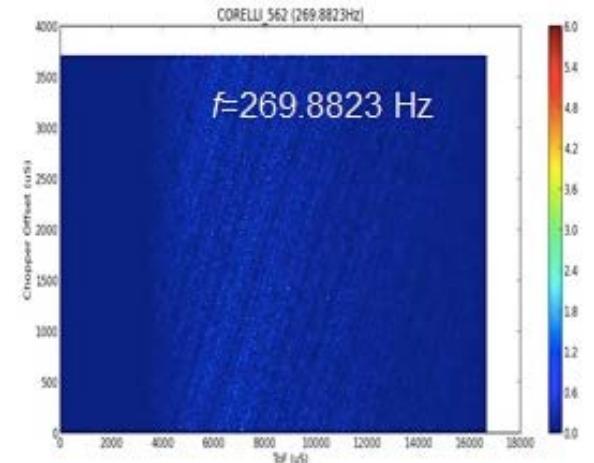
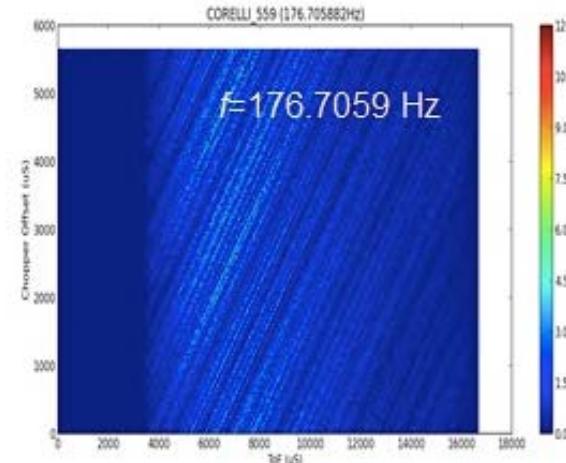
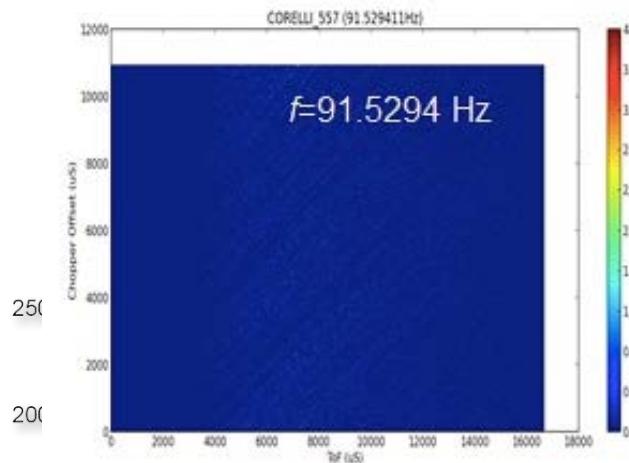
Instrument Proposers

Stephan Rosenkranz
Ray Osborn

Cross Correlation in Action

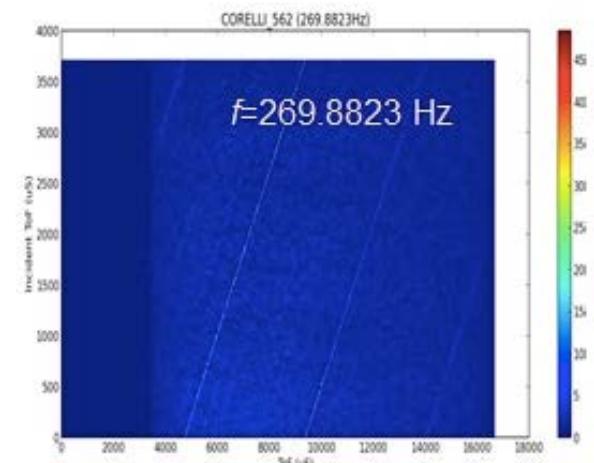
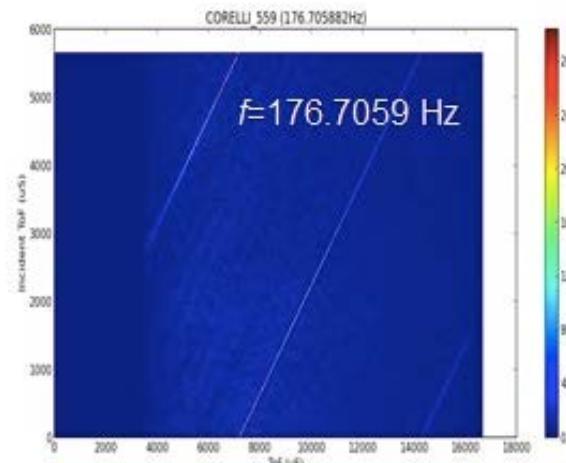
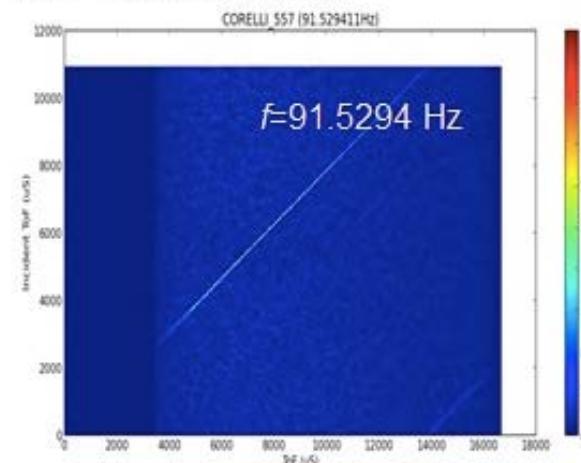


Before cross-correlation



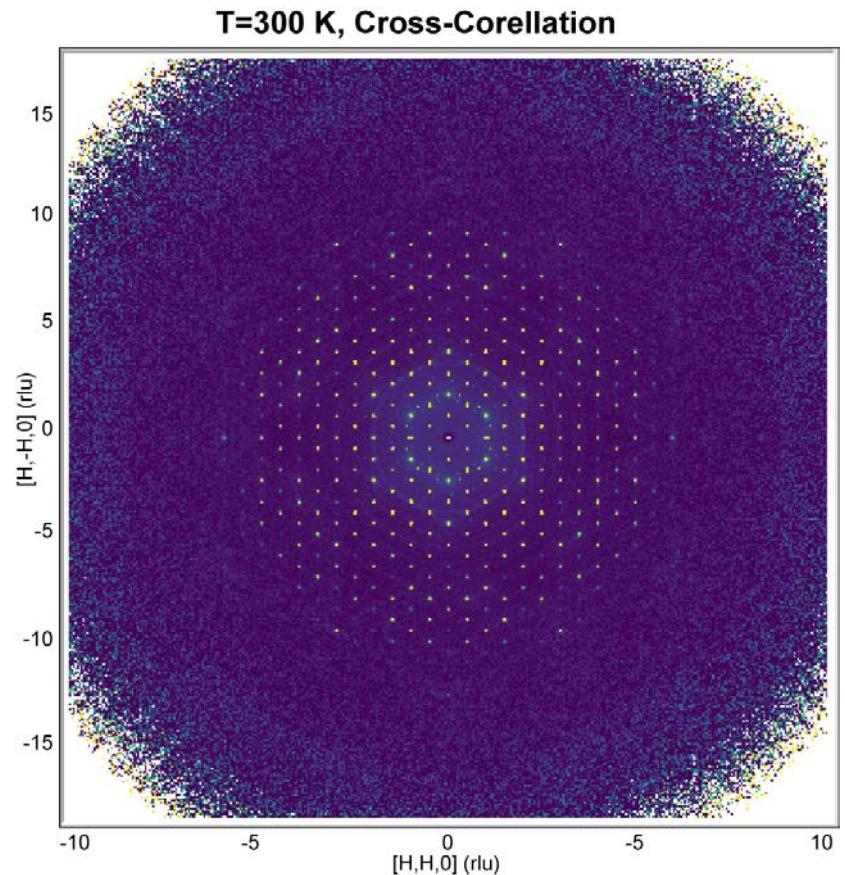
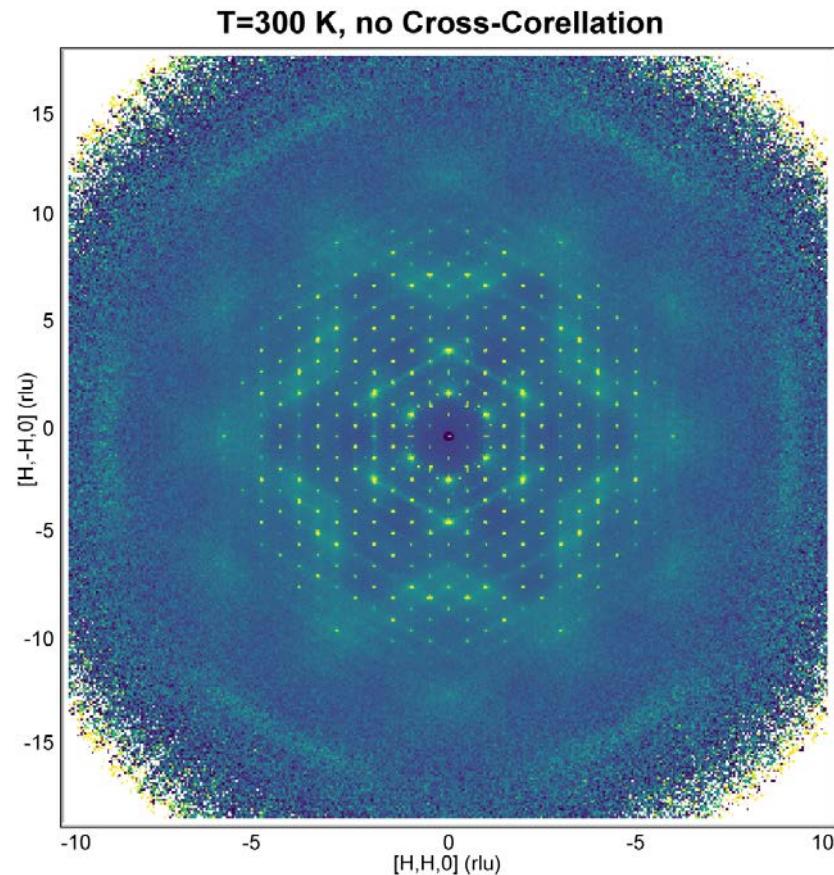
chopper phase

After cross-correlation



Elastic Discrimination with Cross Correlation

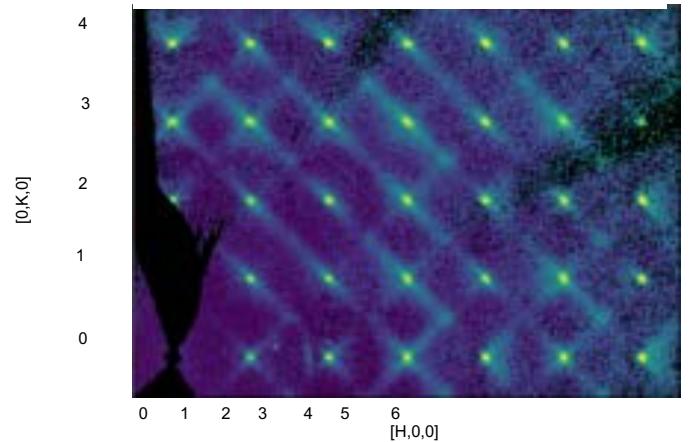
Benzil $C_{14}H_{10}O_2$



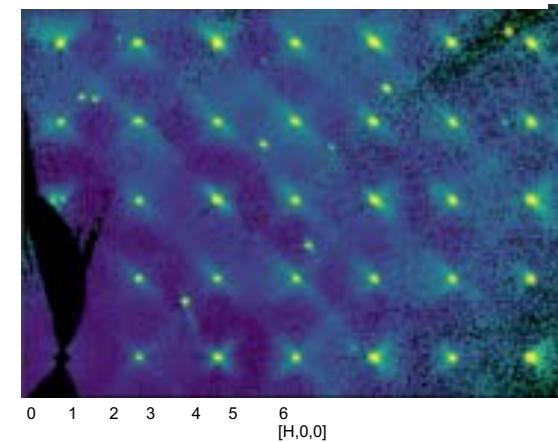
T. R. Welberry and R. Whitfield, Quantum Beam Science **2**, 2 (2018)

Relaxor Ferroelectrics - $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$ - $x\text{PbTiO}_3$

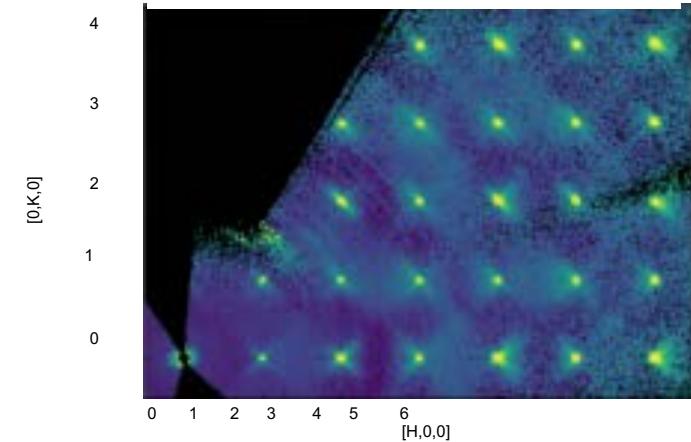
PMN



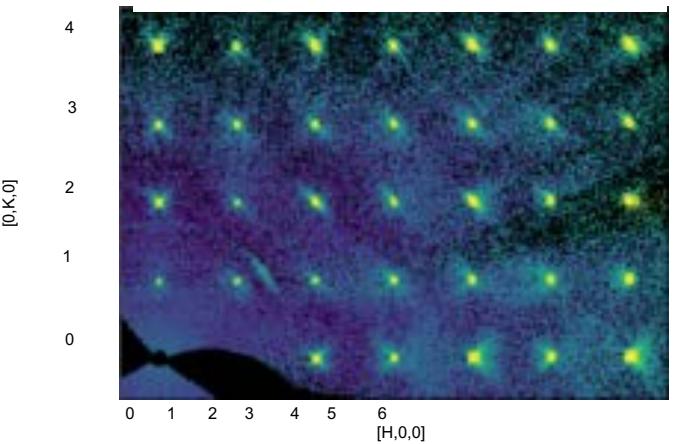
PMN-20PT



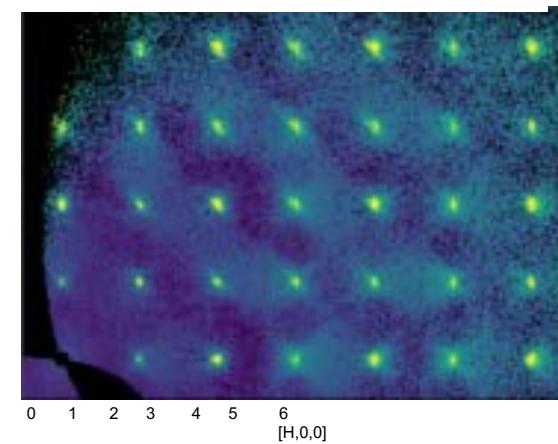
PMN-30PT



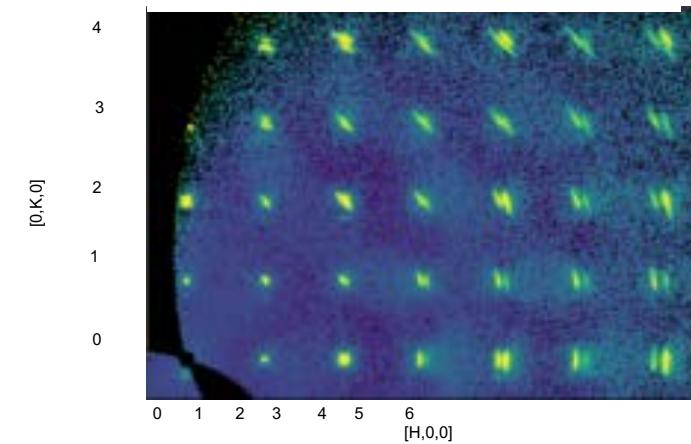
PMN-35PT



PMN-40PT



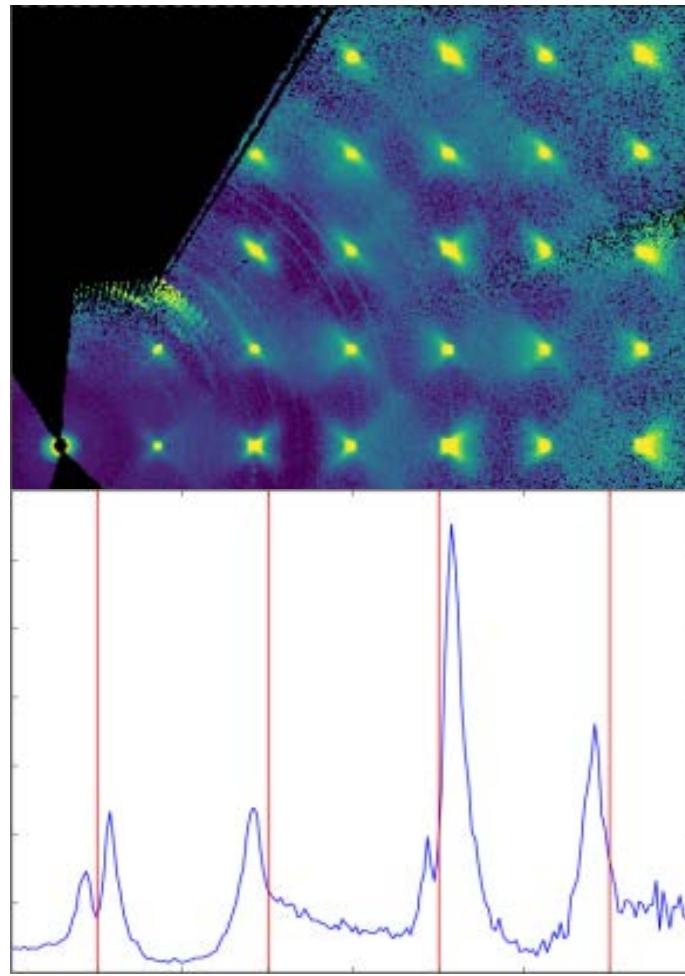
PMN-50PT



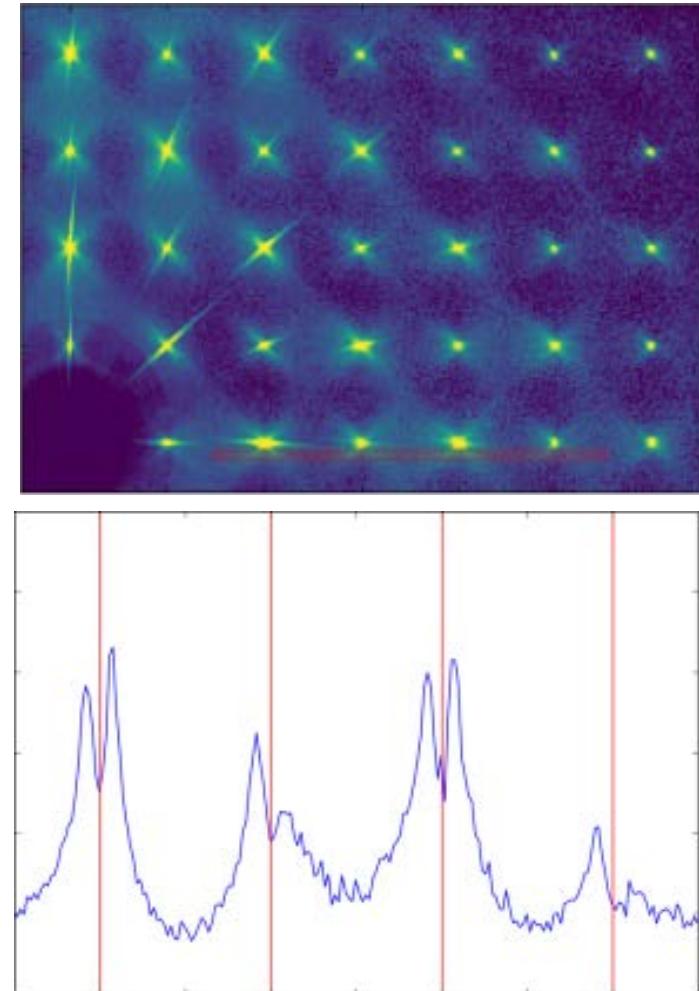
M. J. Krogstad, P. M. Gehring, S. Rosenkranz, R. Osborn, F. Ye, Y. Liu, J. P. C. Ruff, W. Chen, J. M. Wozniak, H. Luo, O. Chmaissem, Z.-G. Ye, and D. Phelan, Nat Mater **48**, 1 (2018).

Complementarity of Neutrons and X-rays

Pb(Mg_{1/3}Nb_{2/3})O₃-30%PbTiO₃



Corelli Neutrons



CHESS 55keV X-rays

Sublattice Melting in Superionic Cu_{1.8}Se

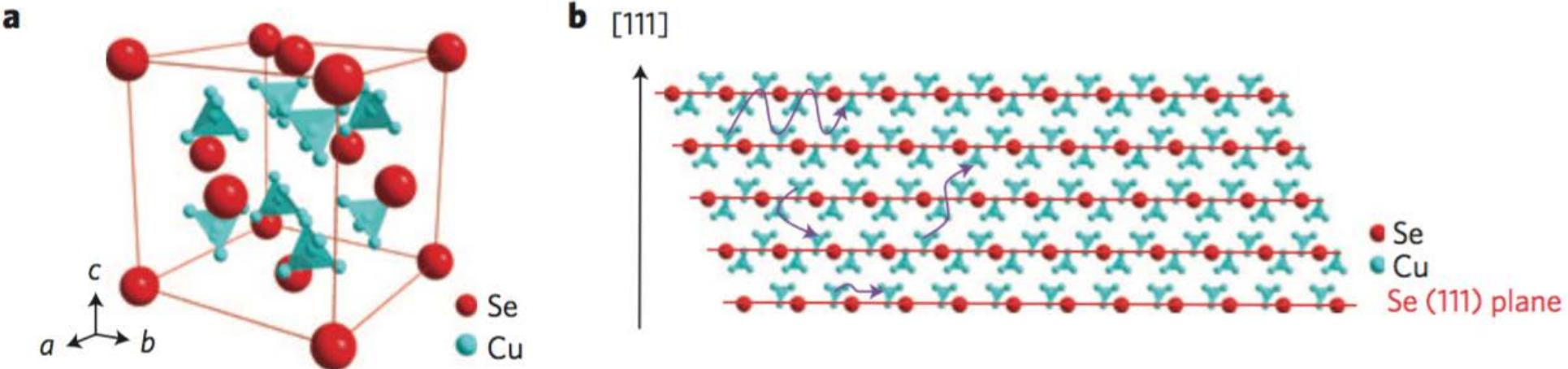
- It has been proposed that Cu_{2-x}Se is a Phonon Liquid-Electron Crystal thermoelectric
 - $zT > 1.5$ at high temperature

LETTERS
PUBLISHED ONLINE: 11 MARCH 2012 | DOI: 10.1038/NMAT3273

nature materials

Copper ion liquid-like thermoelectrics

Huili Liu^{1,2}, Xun Shi^{1,3*}, Fangfang Xu³, Linlin Zhang³, Wenqing Zhang³, Lidong Chen^{1*}, Qiang Li⁴, Ctirad Uher⁵, Tristan Day⁶ and G. Jeffrey Snyder⁶

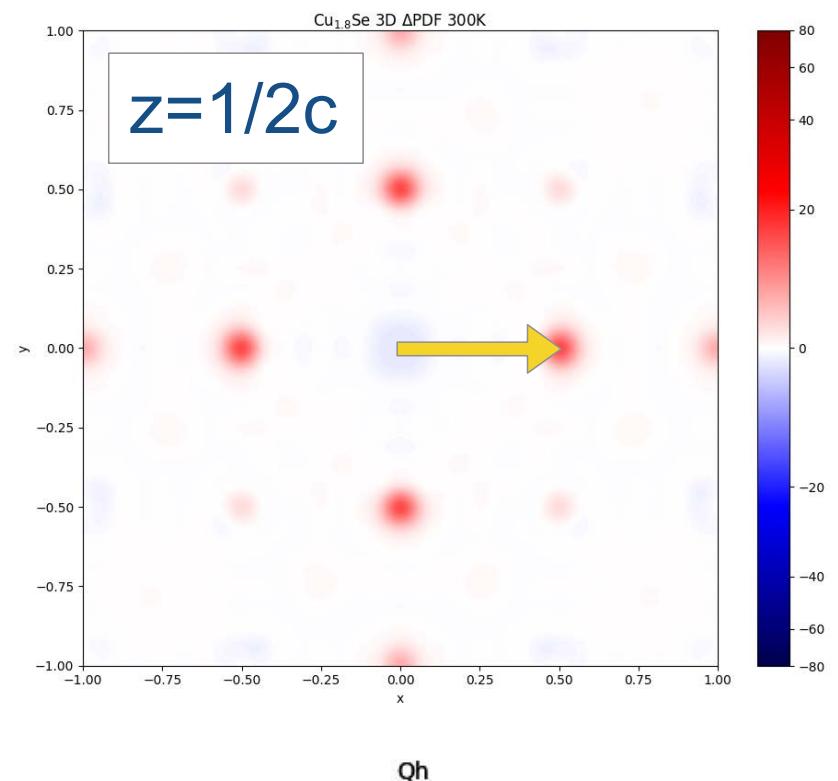
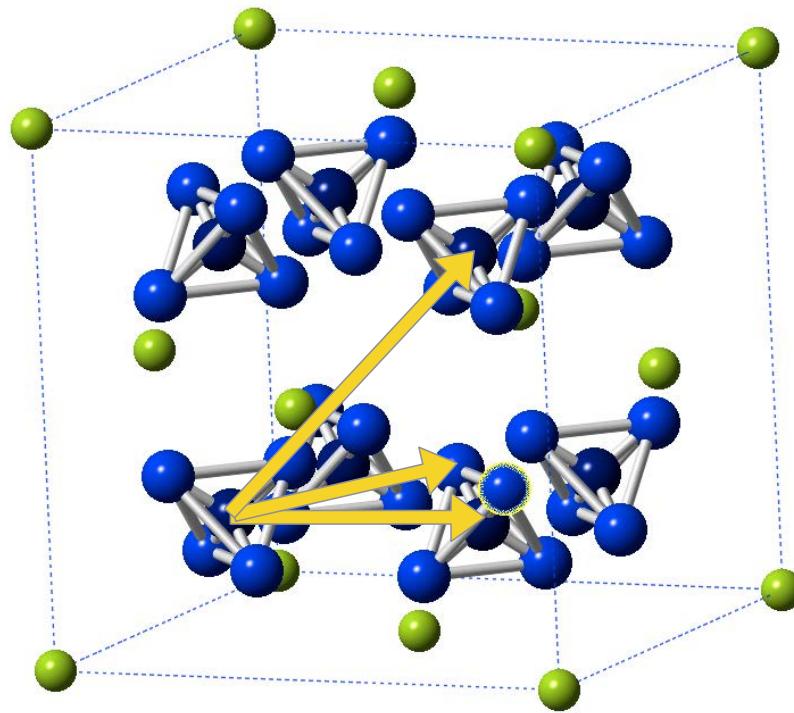


3D- Δ PDF from $\text{Cu}_{1.8}\text{Se}$

Corelli Data



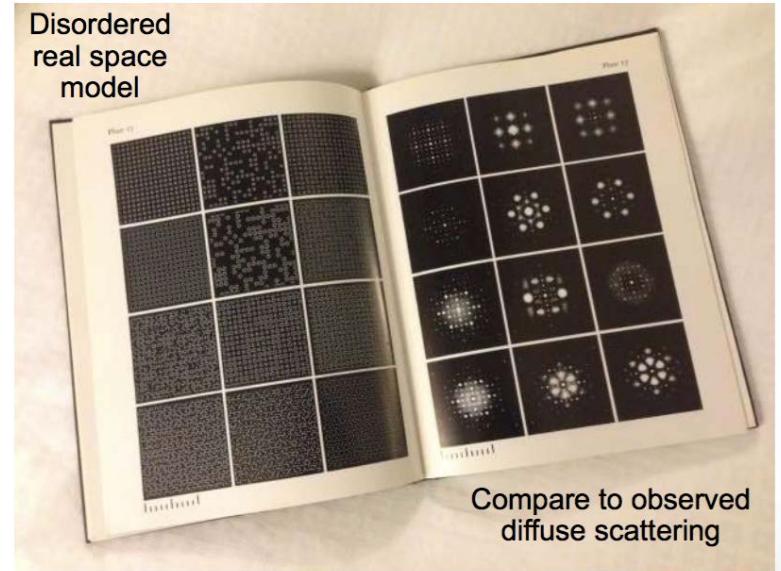
Alex Rettie



Symmetrized Corelli Data

The Future

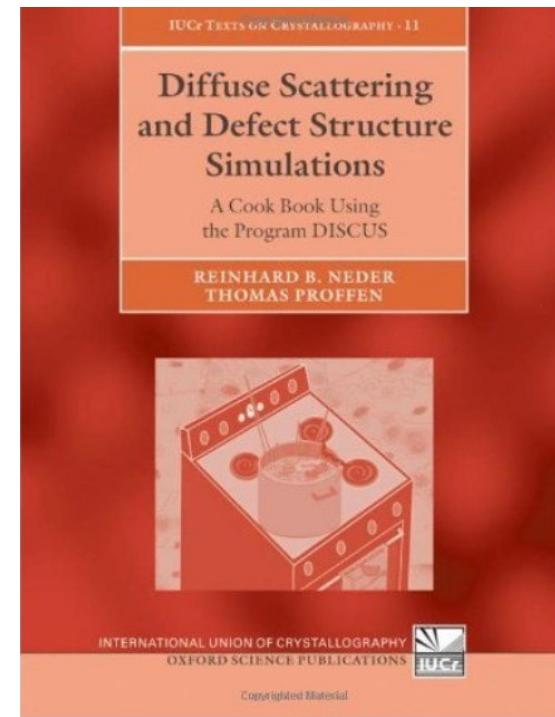
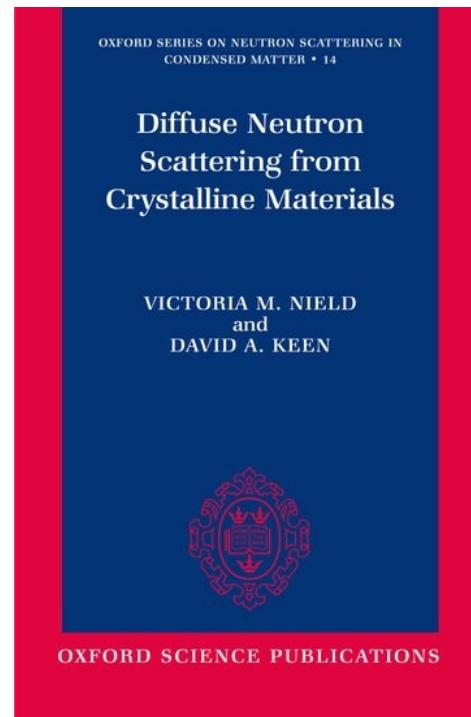
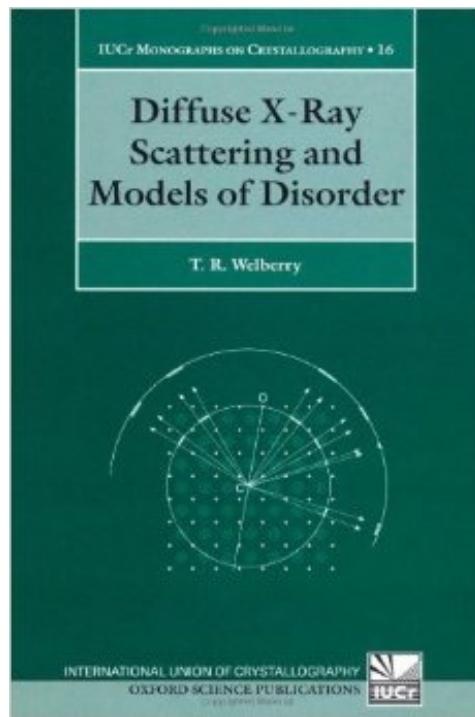
- High-Energy X-rays
 - Absorption lengths similar to neutrons
 - Detectors now have sensors optimized for high energies, e.g. Pilatus 2M CdTe
- Micro-diffuse scattering
 - Benefiting from increased brightness of, e.g., APS Upgrade
- Increasing use of *ab initio* computational modeling
 - Allowing more complex systems to be investigated
 - Less dependence on intuition in modeling
- Enhanced analysis tools
 - Machine learning
 - Correlated data analysis
 - Easier co-refinement of neutrons and x-rays



Atlas of Optical Transforms, Harburn, Taylor and Welberry (1975)

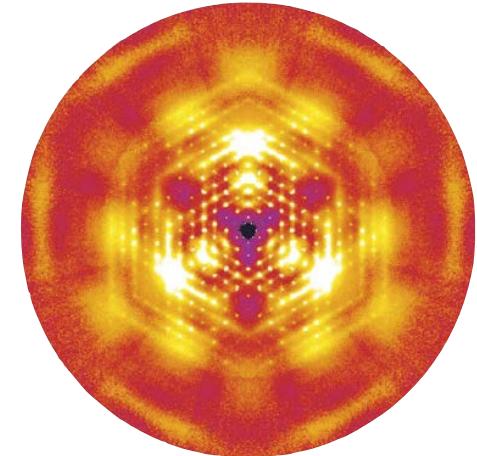
A Few References

- T. R. Welberry & B. Butler, Chem Rev **95**, 2369–2403 (1995).
- F. Frey, Acta Cryst B **51**, 592–603 (1995).
- T. R. Welberry & D. J. Goossens, Acta Cryst A **64**, 23–32 (2007).
- D. A. Keen & A. L. Goodwin, Nature News **521**, 303–309 (2015).

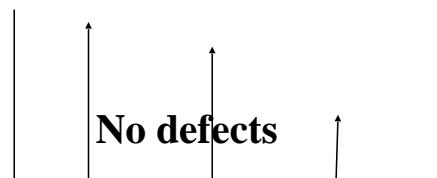


Diffuse Scattering Song

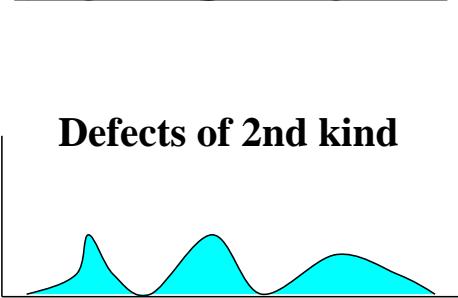
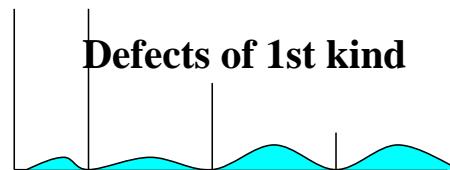
- ▶ Come eager young scholars - so tender and new
I'll teach you diffraction - what I says mostly true
Between the Bragg Peaks lies a world where you see
Fluctuations and defects- they stand out plane-ly
- ▶ *Chorus*
For its dark as a dungeon between the Bragg peaks
But here in the darkness - each defect speaks
It gathers- from throughout- reciprocal space
And re-distributes all over the place.
- ▶ Between the Bragg peaks - one thing that we see
Is TDS on our CCD
Intensity totals are conserved- you can't win
It steals from the Bragg peaks that stay very thin
- ▶ Substitutional alloys can cause quite a stir
The shorter the length scale the greater the blur
With care you can find out the bond length between
Each atom pair type-the measurements clean
- ▶ Dislocations and other- type 2 defects
Destroy the Bragg peaks -they turn them to wrecks
But near the Bragg peaks- you still can see
Intense diffraction continuously
- ▶ Many -are- the defects you find
Between the Bragg peaks where others are blind
So go tell your friends and impress your boss
You've new understanding -with one hours loss



Gene Ice



θ



Krivoglaz Classifications

